Maxima and minima problems

128 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-maxima-and-minima-problems/

Date

A Do Now

Quick questions to start. Some check what you need today; some set a slip that costs marks.

A1 Needed today [3 marks]

Find the coordinates of the stationary points of $y = x^3 - 6x^2 + 9x + 2$ and state the nature of each.

Answer .....

A2 Last lesson [2 marks]

A particle has displacement $s = t^3 - 4t^2 + 5t$ metres at time t seconds. Find the times at which it is at rest.

Answer .....

A3 Needed today [2 marks]

Write $2t^2 - 8t + 11$ in the form $a(t - b)^2 + c$ and state its least value.

Answer .....

A4 Common slip [2 marks]

A particle has velocity $v = t^2 - 6t \text{ m s}^{-1}$ for $0 \leq t \leq 8$. Sam says the greatest speed is 9 m s^{-1}, because $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ when $t = 3$ and $v(3) = -9$. Find the greatest speed.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A particle moves in a straight line from a point O. At time t seconds its displacement from O is s metres, where $s = 12t^2 - 2t^3$, $t \geq 0$. Find the greatest displacement of the particle from O in the positive direction, showing that it is a maximum.

Hint At the greatest displacement the particle is instantaneously at rest, so $v = 0$. Use $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$ to show it is a maximum.

B1 A particle moves in a straight line from a point O. At time t seconds its displacement from O is s metres, where $s = 15t^2 - t^3$, $t \geq 0$. Find the greatest displacement of the particle from O in the positive direction, showing that it is a maximum. [5 marks]

Answer .....

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EXAMPLE 2

A particle moves in a straight line. Its velocity at time t seconds is $v\ \mathrm{m\ s^{-1}}$, where $v = t^2 - 8t + 7$, $0 \leq t \leq 6$. Find the least velocity and the greatest speed of the particle in this interval.

Hint $a = 0$ gives the least velocity. The greatest speed can be at that point (if v is negative there) or at an end of the interval. Compare the sizes.

B2 A particle moves in a straight line. Its velocity at time t seconds is $v\ \mathrm{m\ s^{-1}}$, where $v = 3t^2 - 18t + 15$, $0 \leq t \leq 7$. Find the least velocity and the greatest speed of the particle in this interval. [5 marks]

Answer .....

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EXAMPLE 3

A particle moves in a straight line with velocity $v\ \mathrm{m\ s^{-1}}$ at time t seconds, where $v = 2t^2 - 6t + 7$, $t \geq 0$. Find its least speed, and show that it is never at rest.

Hint Complete the square. A square is never negative.

B3 [4 marks]

A particle moves in a straight line with velocity $v\ \mathrm{m\ s^{-1}}$ at time t seconds, where $v = t^2 - 5t + 8$, $t \geq 0$. Find its least speed, and show that it is never at rest.

Answer .....

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EXAMPLE 4

A drone moves vertically. Its height above the ground at time t seconds is h metres, where $h = \frac{t^4 - 12t^3 + 36t^2}{10} + 20$, $0 \leq t \leq 7$. Find the greatest and least heights of the drone in this interval.

Hint Find where $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$. Then compare h at those times AND at the two ends of the interval.

B4 [6 marks]

A drone moves vertically. Its height above the ground at time t seconds is h metres, where $h = \frac{3t^4 - 20t^3 + 24t^2}{10} + 15$, $0 \leq t \leq 6$. Find the greatest and least heights of the drone in this interval.

Answer .....

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EXAMPLE 5

A train leaves a station A from rest and moves along a straight track towards a signal $400\ \text{m}$ from A. Its distance from A at time t seconds is modelled as $s = 0.6t^2 - 0.01t^3$ metres. Show that, according to the model, the train stops before it reaches the signal. Find its greatest speed.

Hint The train stops when $v = 0$. Compare s at that time with 400. The greatest speed is where $a = 0$.

B5 [5 marks]

A cyclist starts from rest at O and moves along a straight road. Her distance from O at time t seconds is $s = kt^2 - 0.02t^3$ metres, where k is a constant. She comes to rest when she is $80\ \text{m}$ from O. Find the value of k and her greatest speed.

Answer .....

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C1 A particle moves in a straight line with velocity $v = 8t - 2t^2$ m s^{-1}, $0 \le t \le 6$. Leah says the greatest speed is 8 m s^{-1}, because $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0$ at $t = 2$. Find the greatest speed of the particle and explain Leah's mistake. [5 marks]

Answer .....

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C2 A particle moves on a straight line through O. Its displacement from O is $s = t^3 - 12t^2 + 21t$ metres for $0 \leq t \leq 8$. Omar says the greatest distance from O is $10\ \text{m}$, at $t = 1$. Find the greatest distance of the particle from O. [5 marks]

Answer .....

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C3 A particle moves in a straight line with velocity $v = -3t^2 + 18t + 21\ \mathrm{m\ s^{-1}}$, $t \geq 0$. Find the greatest velocity of the particle, showing that it is a maximum, and find the time at which the particle comes to rest. [6 marks]

Answer .....

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C4 A ball is thrown upwards. Its height above the ground at time t seconds is modelled as $h = 4.5t^2 - 0.5t^3$ metres. Tom says: "The greatest height is when the acceleration is zero, so $9 - 3t = 0$, $t = 3$ and the greatest height is $27\ \text{m}$." Explain Tom's mistake and find the greatest height. [5 marks]

Answer .....

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C5 A particle moves on the x-axis with displacement $x = 2t^3 - 0.25t^4$ [6 marks] metres at time t seconds, $t \geq 0$. Find the greatest velocity of the particle and the greatest acceleration of the particle.

Answer .....

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D Exam-style questions

Edexcel exam style

D1 A train leaves a station A from rest and moves along a straight track to the next station B, where it stops. Until it stops at B, its distance from A at time t seconds is modelled as s metres, where $s = 0.9t^2 - 0.005t^3$.

(a) Find the distance from A to B. [3 marks]

Answer .....

(b) Find the greatest speed of the train. [3 marks]

Answer .....

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(c) The speed limit on this track is $45\ \text{m s}^{-1}$. Find, according to the model, the length of time for which the train is travelling faster than the speed limit. [4 marks]

Answer .....

(d) Explain why the model is not valid for $t > 120$. [2 marks]

Total for D1: 12 marks

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D2

> Ria finds the least speed like this: "$a = 6t - 24 = 0$ at $t = 4$, and $v(4) = -12$, so the least speed of P is $12\ \text{m s}^{-1}$."

A particle P moves in a straight line. At time t seconds its velocity is $v\ \text{m s}^{-1}$, where $v = 3t^2 - 24t + 36$, $0 \leq t \leq 5$.

(a) Explain the mistake in Ria's working. [2 marks]

(b) Find the greatest speed of P. [3 marks]

Answer .....

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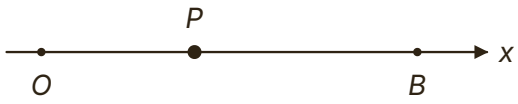
(c) Find all the times at which the speed of P is $9\ \mathrm{m\ s^{-1}}$. [4 marks]

Answer .....

Total for D2: 9 marks

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D3 A particle P moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = t^3 - 12t^2 + 45t$, $0 \leq t \leq 6$. The point B on the x-axis has $x = 60$. Find the least distance between P and B during the motion, and the times at which it occurs. [8 marks]



Answer .....

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D4 A particle P moves on a straight line through O. At time t seconds its displacement from O is s metres, where $s = pt^3 + qt^2 + 12t$ and p and q are constants. P is instantaneously at rest when $t = 2$ and when $t = 3$.

(a) Find the values of p and q. [4 marks]

Answer .....

(b) Find the displacement of P at each time it is at rest, and state whether each is a greatest or a least displacement near that time. [3 marks]

Answer .....

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(c) Find the least velocity of P. [2 marks]

Answer .....

Total for D4: 9 marks

D5 A lift moves vertically upwards from rest at the ground floor. At time t seconds its velocity is $v \text{ m s}^{-1}$, where $v = 3t - 0.3t^2$, $0 \leq t \leq 10$. A passenger of mass 60 kg stands on the floor of the lift.

(a) Show that the lift is at rest when $t = 10$. [2 marks]

Answer .....

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(b) Find the greatest speed of the lift. [3 marks]

Answer .....

(c) The height of the lift above the ground floor is $h = 1.5t^2 - 0.1t^3$ metres. Show that this agrees with the velocity, and find the height through which the lift rises in the 10 seconds. [2 marks]

Answer .....

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(d) Find the greatest acceleration of the lift and the force exerted by the floor on the passenger at that instant. [3 marks]

Answer .....

Total for D5: 10 marks

D6 A particle P moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = t^3 - 3t^2 - 9t + 4$, $0 \leq t \leq 5$.

(a) Find the least value of x in this interval. [3 marks]

Answer .....

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(b) Find the greatest distance of P from O in this interval. [3 marks]

Answer .....

(c) Find the greatest speed of P in this interval. [3 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 A car starts from rest at a point A and moves along a straight road. Its distance from A at time t seconds is modelled as $s = pt^2 - qt^3$ metres, where p and q are positive constants. The car comes to rest at a point B, $300\ \text{m}$ from A, when $t = 20$. Find p and q, the greatest speed of the car, and the length of time for which its speed is greater than $18\ \text{m s}^{-1}$. [10 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write down v. "Comes to rest at B when $t = 20$" gives two equations: $v(20) = 0$ and $s(20) = 300$.

Hint 2 The greatest speed is where $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0$. Check that v is positive there.

Hint 3 Solve $v > 18$ as a quadratic inequality. The length of time is the difference between its two roots.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the greatest displacement or velocity by setting the derivative to zero and checking its nature.	○ ○ ○
I can find the greatest speed on an interval, checking both directions and the end points.	○ ○ ○
I can find a least speed by completing the square and show a particle is never at rest.	○ ○ ○
I can use a given greatest value to find a constant in a model.	○ ○ ○

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