Models, assumptions and units

126 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-models-assumptions-units/

Date

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A Do Now

Quick questions to start. Some use skills from GCSE and Pure Year 1; some set slips that cost marks.

A1 Needed today [2 marks]

Solve $4x^2 - 13x - 12 = 0$.

Answer .....

A2 Needed today [3 marks]

Write $10 + 8t - 2t^2$ in the form $a - b(t - c)^2$. Hence write down its greatest value and the value of t where it happens.

Answer .....

A3 Common slip [3 marks]

Convert $54\ \text{km h}^{-1}$ to m s^{-1} and $2.4\ \text{g cm}^{-3}$ to kg m^{-3}.

Answer .....

A4 Common slip [2 marks]

A bag has a mass of $6\ \text{kg}$. Leah says that the weight of the bag is $6\ \text{kg}$. Explain why she is wrong and find the weight of the bag.

Answer .....

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A5 Needed today [2 marks]

Write $0.00036\ \text{km}$ and $4.5 \times 10^4\ \text{cm}$ in metres, in standard form.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A ball is thrown. Its height h m above the ground when it has travelled a horizontal distance of x m is modelled by $h = 1.8 + 0.6x - 0.05x^2$.

(a) Find the height from which the ball is thrown.

(b) Find the height of the ball when $x = 4$.

(c) Use the model to predict the height of the ball when $x = 20$, and comment on the validity of this prediction.

Hint The start of the motion is at $x = 0$. A model is only valid while its answers make sense in the context.

B1 [5 marks]

A stone is thrown from the top of a cliff into the sea. Its height h m above sea level t seconds after it is thrown is modelled by $h = 30 + 7t - 4.9t^2$.

(a) Write down the height of the cliff.

(b) Find the height of the stone when $t = 2$.

(c) Use the model to predict the height of the stone when $t = 5$, and comment on the validity of this prediction.

Answer .....

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EXAMPLE 2

The height h m of a ball above the ground, when it has travelled a horizontal distance x m, is modelled by $h = 1.6 + 1.2x - 0.1x^2$, from when it is hit until it lands.

(a) Find the range of values of x for which the model is valid.

(b) Find the greatest height of the ball and the value of x where it happens.

(c) Find the two values of x at which the ball is 4.8 m above the ground.

Hint Set $h = 0$ for the landing point. Reject a root that is before the motion starts. Completing the square gives the greatest value.

B2 The height h m of a ball above the ground, when it has travelled a horizontal distance x m, is modelled by $h = 2 + 0.9x - 0.05x^2$, from when it is hit until it lands.

(a) Find the range of values of x for which the model is valid. [3 marks]

Answer .....

(b) Find the greatest height of the ball. [2 marks]

Answer .....

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(c) Find the two values of x at which the ball is 5.6 m above the ground. [2 marks]

Answer .....

Total for B2: 7 marks

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EXAMPLE 3

The height h m of a ball t seconds after it is thrown upwards is modelled by $h = a + bt - 4.9t^2$, where a and b are constants. The ball is 11.8 m high when $t = 1$ and again when $t = 2$.

(a) Find a and b.

(b) Find the greatest height of the ball.

Hint Each piece of data gives one equation. Two unknowns need two equations: solve them simultaneously.

B3 The height h m of a ball t seconds after it is thrown upwards is modelled by $h = a + bt - 4.9t^2$, where a and b are constants. When $t = 1$, $h = 19.6$ and when $t = 2$, $h = 24.5$. Find a and b, and find the time at which the ball reaches the ground. [6 marks]

Answer .....

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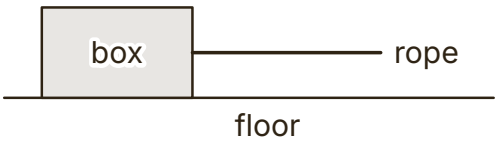
EXAMPLE 4

A box is pulled across a horizontal floor by a rope, as shown in the diagram.

(a) Draw a force diagram for the box and name each force.

(b) State the effect on any calculations of each assumption: (i) the box is modelled as a particle, (ii) the rope is light, (iii) the rope is inextensible, (iv) the floor is smooth.

(c) Suggest one refinement to the model and say how it changes the predicted motion.



Hint Say what each assumption lets you ignore, in the context of the question. A refinement adds back something that was ignored.

B4 [4 marks]

A tennis ball is hit across a court. In a model, the ball is a particle and air resistance is negligible.

(a) State the effect of modelling the ball as a particle.

(b) State the effect of the assumption that air resistance is negligible.

(c) Explain whether the model is likely to overestimate or underestimate how far the ball travels.

(d) Suggest one refinement to the model.

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EXAMPLE 5

Write each quantity in SI units.

(a) $72\ \text{km h}^{-1}$

(b) $0.8\ \text{g cm}^{-3}$

(c) $2.5 \times 10^5\ \text{cm}$ per minute

(d) the weight of a mass of 450 g

Hint SI units: kg, m, s, and N for force. Convert each unit in turn. Weight needs the mass in kg first.

B5 Write each quantity in SI units. [5 marks]

(a) $126\ \text{km h}^{-1}$

(b) $13.6\ \text{g cm}^{-3}$

(c) 3.6×10^4 km per day

(d) the weight of a mass of 750 g

Answer .....

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C1 Kai converts $90\ \text{km h}^{-1}$ to m s^{-1} and writes $90 \times 3.6 = 324\ \text{m s}^{-1}$. [4 marks]
Explain his mistake. A car travels at a constant $90\ \text{km h}^{-1}$. Find the time it takes to travel 1.5 km.

Answer .....

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C2 A stone is thrown upwards from the top of a tower. Its height h m above the ground t seconds later is modelled by $h = 20 + 15t - 4.9t^2$ until it hits the ground. Rhys solves $h = 0$, takes the root $t = -1.00$, and says the model is valid for all $t \geq 0$. Find the time at which the stone hits the ground and the range of values of t for which the model is valid. Explain both of his errors. [5 marks]

Answer .....

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C3 A ball is thrown and modelled as a particle moving under gravity only, with $g = 9.8\ \text{m s}^{-2}$. Three students state a limitation of the model. [4 marks]

Ana: "Air resistance limits the height."

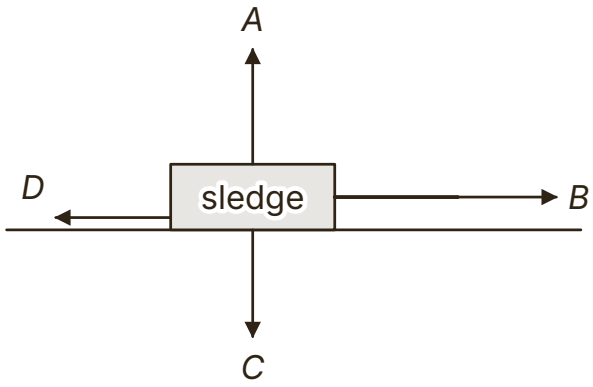
Ben: "As a particle, the mass of the ball is ignored."

Cal: "Air resistance is ignored. It acts against the motion and slows the ball, so the real ball does not go as high as the model predicts."

Say which answer earns the mark, and what is wrong with each of the other two.

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C4 A sledge is pulled across horizontal snow by a horizontal rope. [4 marks]
The diagram shows the four forces acting on the sledge.
(a) Name the forces A, B, C and D.
(b) The snow is now modelled as smooth. State which force is removed from the diagram.



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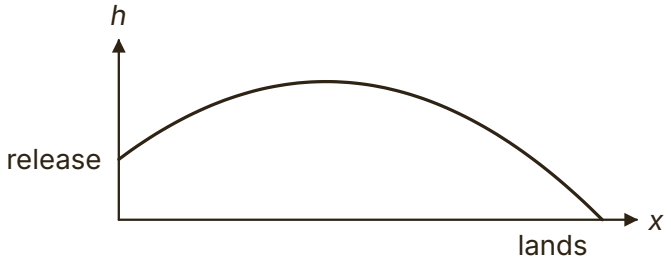
C5 A ball is thrown upwards. A model for its height is $h = 1.2 + 9.8t - 4.9t^2$, where h m is the height after t seconds. The measured heights at $t = 0.5$, 1.0 and 1.5 are 4.7 m, 5.8 m and 4.3 m. Find the heights the model predicts at these times. Comment on how well the model fits the data, give a reason for the difference, and suggest a refinement. [5 marks]

Answer .....

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D1 A shot is thrown. Its height h m above the ground when it has travelled a horizontal distance x m is modelled by $h = 1.75 + 0.75x - 0.0625x^2$, from when it is released until it lands. The shot is modelled as a particle.

(a) Write down the height at which the shot is released, and find its height when $x = 10$. [2 marks]



Answer .....

(b) Find the horizontal distance from the point of release to the point where the shot lands. [3 marks]

Answer .....

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(c) By completing the square, show that the greatest height of the shot is 4 m, and find the value of x at which it happens. [3 marks]

Answer .....

(d) State the range of values of x for which the model is valid, and state one limitation of the model. [2 marks]

Total for D1: 10 marks

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D2

A stone is thrown upwards from a bridge. Its height h m above the water t seconds later is modelled by $h = 12 + 9.8t - 4.9t^2$, until it reaches the water. Jay says: "The stone is 12 m above the water at $t = 0$ and again at $t = 2$. So the path is symmetric about $t = 2$, and the stone reaches the water at $t = 4$."

Jay is wrong.

(a) Show that the stone is 12 m above the water when $t = 2$. [2 marks]

Answer .....

(b) Explain why Jay is wrong. [2 marks]

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(c) Find the time at which the stone reaches the water. [3 marks]

Answer .....

(d) Find the length of time for which the stone is more than 15 m above the water. [3 marks]

Answer .....

Total for D2: 10 marks

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D3 A toy rocket is launched from the ground. Its height h m above the ground t seconds after launch is modelled by $h = at - bt^2$, where a and b are constants. The rocket is 36 m high when $t = 2$ and 45 m high when $t = 5$. Find the length of time for which the rocket is more than 36 m above the ground. [7 marks]

Answer .....

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D4 The stopping distance d m of a car travelling at $v\ \mathrm{m\ s^{-1}}$ is modelled by $d = pv + qv^2$, where p and q are constants. On a dry road, at $10\ \mathrm{m\ s^{-1}}$ the stopping distance is 15 m, and at $20\ \mathrm{m\ s^{-1}}$ it is 40 m.

(a) Find p and q. [4 marks]

Answer .....

(b) The speed limit on a road is $108\ \mathrm{km\ h^{-1}}$. Use the model to find the stopping distance at this speed. [3 marks]

Answer .....

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(c) A driver sees a hazard 55 m ahead. Find, in $\mathrm{km\ h^{-1}}$, the greatest speed at which she can stop before the hazard. [3 marks]

Answer .....

(d) Explain why using the model on a wet road may give a stopping distance that is too small. [2 marks]

Total for D4: 12 marks

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D5 A crate of mass 350 kg hangs at rest from a crane by a cable. The cable is modelled as light and inextensible, and the crate as a particle.

(a) State what is meant by modelling the cable as light, and as inextensible. [2 marks]

(b) Find the weight of the crate. [2 marks]

Answer .....

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(c) While the crate is at rest, the tension in the cable equals the total weight it supports. The cable can safely support a tension of 3.5 kN. A load of mass m kg is placed on the crate. Find the greatest value of m, to the nearest kilogram, for which the cable is safe. [3 marks]

Answer .....

(d) Give one reason why the tension in the real cable could be greater than the total weight of the crate and its load. [2 marks]

Total for D5: 9 marks

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D6 Oil has a density of $0.92\ \mathrm{g\ cm^{-3}}$. A drum holds $0.21\ \mathrm{m^3}$ of oil.

(a) Write the density of the oil in $\mathrm{kg\ m^{-3}}$. [2 marks]

Answer .....

(b) Find the mass and the weight of the oil in a full drum. [3 marks]

Answer .....

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(c) A van can carry a load of mass at most 1.5 tonnes. Ignoring the mass of the drums themselves, find the greatest number of full drums of oil the van can carry. [2 marks]

Answer .....

Total for D6: 7 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 A volleyball is served from a point O. Its height h m above the court, when it has travelled a horizontal distance x m, is modelled by $h = 2.4 + kx - 0.05x^2$, where k is a constant. The top of the net is 2.43 m above the court at $x = 9$. The serve is good if the ball passes over the net and then lands on the court, which ends at $x = 18$. Find the set of values of k for which the serve is good. For $k = 0.6$, find where the ball lands and its greatest height. State one limitation of the model. [10 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the two conditions as inequalities: one at $x = 9$ and one at $x = 18$.

Hint 2 Passing over the net needs $h(9) > 2.43$. Landing on the court needs $h(18) \leq 0$. Solve each for k and combine.

Hint 3 For $k = 0.6$, set $h = 0$ and take the positive root. Complete the square, $h = 4.2 - 0.05(x - 6)^2$, for the greatest height.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use a model to make a prediction and say whether the prediction is valid.	○ ○ ○
I can find the constants in a model from data and the range of values for which the model is valid.	○ ○ ○
I can state the effect of a modelling assumption, a limitation of a model and a refinement.	○ ○ ○
I can convert quantities to SI units and find a weight using W = mg.	○ ○ ○

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