Motion in two dimensions

125 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-motion-in-two-dimensions/

Date

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks.

A1 Last lesson [2 marks]

Find the resultant of the forces $(3\mathbf{i} - 2\mathbf{j})$ N and $(-\mathbf{i} + 6\mathbf{j})$ N, and its magnitude.

Answer .....

A2 Needed today [2 marks]

A particle starts at the point with position vector $(\mathbf{i} + 4\mathbf{j})$ m and moves with constant velocity $(2\mathbf{i} - 3\mathbf{j})\ \text{m s}^{-1}$. Find its position vector after 5 s.

Answer .....

A3 Needed today [3 marks]

A particle moves in a straight line with initial velocity $5\ \text{m s}^{-1}$ and constant acceleration $-2\ \text{m s}^{-2}$. Find the time when it is instantaneously at rest and its displacement then.

Answer .....

A4 Common slip [2 marks]

A particle of mass 2 kg has acceleration $(3\mathbf{i} - 4\mathbf{j})\ \text{m s}^{-2}$. Find the magnitude of the resultant force on it.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A particle of mass 4 kg is acted on by the forces $(6\mathbf{i} - 2\mathbf{j})$ N and $(2\mathbf{i} + 8\mathbf{j})$ N. Find the acceleration of the particle, its magnitude, and the bearing on which it acts.

Hint Add the forces first, then divide the resultant by the mass. The acceleration is in the same direction as the resultant force.

B1 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A particle of mass 0.5 kg is acted on by the forces $(3\mathbf{i} + \mathbf{j})$ N and $(-5\mathbf{i} + 2\mathbf{j})$ N. Find the acceleration of the particle, its magnitude, and the bearing on which it acts. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2

A particle of mass 2 kg moves under two forces, $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N. The acceleration of the particle is $(4\mathbf{i} + \mathbf{j})\ \text{m s}^{-2}$. Find p and q.

Hint $\mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$ is two equations: one for the $\mathbf{i}$ parts and one for the $\mathbf{j}$ parts.

B2 A particle of mass m kg moves under the forces $(5\mathbf{i} + 3\mathbf{j})$ N, $(-4\mathbf{i} + 2\mathbf{j})$ N and $(p\mathbf{i} - \mathbf{j})$ N. Its acceleration has magnitude $2.5\ \text{m s}^{-2}$ and acts in the direction of $3\mathbf{i} + 4\mathbf{j}$. Find m and p. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 3

A particle of mass 2 kg is at rest at the point with position vector $(\mathbf{i} - 2\mathbf{j})$ m. A constant force $(4\mathbf{i} + 6\mathbf{j})$ N then acts on it. Find its velocity and its speed after 4 s, and its position vector then.

Hint Find $\mathbf{a}$ first. The vector equations work exactly like the one-dimensional ones. Speed is the magnitude of the velocity vector.

B3 A particle of mass 0.5 kg passes through the origin O with velocity $(3\mathbf{i} - 2\mathbf{j})\ \mathrm{m\ s^{-1}}$. A constant force $(\mathbf{i} + 2\mathbf{j})$ N acts on it. Find its speed 3 s later and its distance from O then. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4

A particle has initial velocity $(2\mathbf{i} - 5\mathbf{j})\ \mathrm{m\ s^{-1}}$ and constant acceleration $(\mathbf{i} + 2\mathbf{j})\ \mathrm{m\ s^{-2}}$. Find the time when it moves parallel to $\mathbf{i}$, and the time when it moves parallel to $\mathbf{i} - \mathbf{j}$.

Hint Write $\mathbf{v}$ at time t. Parallel to $\mathbf{i}$ means the $\mathbf{j}$ part is zero. Parallel to $\mathbf{i} - \mathbf{j}$ means the $\mathbf{j}$ part is minus the $\mathbf{i}$ part.

B4 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A particle has initial velocity $(-5\mathbf{i} + 8\mathbf{j})\ \mathrm{m\ s^{-1}}$ and constant acceleration $(3\mathbf{i} - \mathbf{j})\ \mathrm{m\ s^{-2}}$. Find the time when it moves parallel to $4\mathbf{i} + 5\mathbf{j}$ and its speed then. Find the time when it moves due east. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 5

A particle of mass 2 kg is at O with velocity $(4\mathbf{i} - 2\mathbf{j})\ \text{m s}^{-1}$. It moves under a constant force $\mathbf{F}$. Two seconds later it is at the point with position vector $(6\mathbf{i} + 2\mathbf{j})$ m. Find $\mathbf{F}$ and its magnitude.

Hint Put the known values into $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ and solve for $\mathbf{a}$. Then $\mathbf{F} = m\mathbf{a}$.

B5 A particle of mass 3 kg is at the point with position vector $(\mathbf{i} + 2\mathbf{j})$ m with velocity $(2\mathbf{i} + \mathbf{j})\ \text{m s}^{-1}$. It moves under a constant force $\mathbf{F}$. Four seconds later it is at the point with position vector $(5\mathbf{i} + 14\mathbf{j})$ m. Find $\mathbf{F}$ and its magnitude. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 A particle has initial velocity $(5\mathbf{i} - 2\mathbf{j})\ \text{m s}^{-1}$ and constant acceleration $(-\mathbf{i} + 3\mathbf{j})\ \text{m s}^{-2}$. Leo finds the velocity after 4 s correctly and then writes: "The speed is $1 + 10 = 11\ \text{m s}^{-1}$." Find the velocity after 4 s, explain Leo's mistake, and find the speed. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C2 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A particle of mass 5 kg moves under a resultant force $(-10\mathbf{i} - 24\mathbf{j})$ N. Find its acceleration and the magnitude of the acceleration. Mia says: "$\tan^{-1}\dfrac{4.8}{2} = 67.4°$, so the acceleration acts on a bearing of $067°$." Explain why Mia is wrong and find the bearing. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C3 A particle starts at O with velocity $6\mathbf{i}\ \text{m s}^{-1}$ and has constant acceleration $\mathbf{j}\ \text{m s}^{-2}$. Ken says: "In 4 s it moves 24 m across and 8 m up, so it is 32 m from O." Find the position vector after 4 s, explain why Ken is wrong, and find the distance from O. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C4 A particle of mass 2 kg is at rest in equilibrium under three forces. One of them is $(6\mathbf{i} - 8\mathbf{j})$ N. This force is removed and the other two stay the same. Find the acceleration of the particle and the distance it moves in the next 3 s. [4 marks]

Answer .....

C5 A particle moves from rest under two forces, $(3\mathbf{i} - \mathbf{j})$ N and $(5\mathbf{i} + 7\mathbf{j})$ N. Its acceleration has magnitude $2.5\ \text{m s}^{-2}$. Find the mass of the particle and the time it takes to move 20 m. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D1 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A boat of mass 400 kg is modelled as a particle. Its engine gives a force of $(500\mathbf{i} + 300\mathbf{j})$ N and the wind gives a force of $(100\mathbf{i} + 300\mathbf{j})$ N. All other forces are ignored. At time $t = 0$ the velocity of the boat is $(2\mathbf{i} - 3\mathbf{j})\ \mathrm{m\ s^{-1}}$.

(a) Find the acceleration of the boat. [2 marks]

Answer .....

(b) Find the speed of the boat at $t = 4$ s. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the time when the boat is moving due east. [2 marks]

Answer .....

(d) Find the distance of the boat from its position at $t = 0$ at the time found in part (c). [3 marks]

Answer .....

Total for D1: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D2

A particle of mass 0.5 kg starts from rest at O and moves under a constant force. At $t = 4$ s it is at the point with position vector $(12\mathbf{i} - 16\mathbf{j})$ m. Sam writes: "It moves $12\mathbf{i} - 16\mathbf{j}$ in 4 s, so at $t = 4$ its velocity is $\frac{1}{4}(12\mathbf{i} - 16\mathbf{j}) = (3\mathbf{i} - 4\mathbf{j})\ \text{m s}^{-1}$."

Sam is wrong.

(a) Explain Sam's mistake. [2 marks]

(b) Find the force acting on the particle and its magnitude. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the speed of the particle at $t = 4$ s. [2 marks]

Answer .....

(d) At $t = 4$ s the force is replaced by a constant force $(-1.5\mathbf{i} + 2\mathbf{j})$ N. Find how long after $t = 4$ s the particle is instantaneously at rest. [3 marks]

Answer .....

Total for D2: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

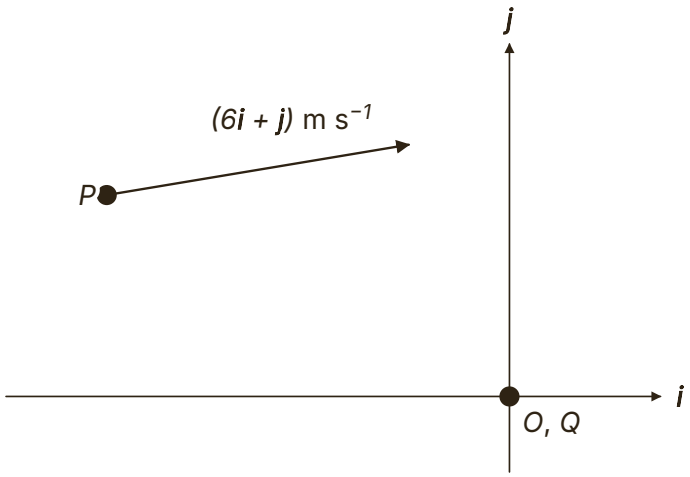
D3 A particle of mass 0.5 kg starts from rest and moves under two constant forces, $(2\mathbf{i} + 3\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N. After 2 s its velocity is $(20\mathbf{i} - 4\mathbf{j})\ \text{m s}^{-1}$. Find p and q, and find the distance the particle moves in these 2 s. [7 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D4 A particle Q of mass 2 kg is at rest at the origin O at time $t = 0$. A constant force $(4\mathbf{i} + 2\mathbf{j})$ N then acts on Q. A second particle P moves with constant velocity $(6\mathbf{i} + \mathbf{j})\ \mathrm{m\ s^{-1}}$, and at $t = 0$ it is at the point with position vector $(-8\mathbf{i} + 4\mathbf{j})$ m.

(a) Find the position vector of Q at time t seconds. [3 marks]



Answer .....

(b) Write down the position vector of P at time t seconds. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Show that P and Q collide, and find the time and the position vector of the collision. [4 marks]

Answer .....

(d) Find the speed of Q when the particles collide. [2 marks]

Answer .....

Total for D4: 11 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D5 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A particle P of mass 2 kg moves with constant velocity $(\mathbf{i} + 4\mathbf{j})\ \mathrm{m\ s^{-1}}$ under the forces $\mathbf{F}_1 = (4\mathbf{i} - 3\mathbf{j})$ N, $\mathbf{F}_2 = (p\mathbf{i} + 6\mathbf{j})$ N and $\mathbf{F}_3 = (2\mathbf{i} + q\mathbf{j})$ N.

(a) Find the value of p and the value of q. [2 marks]

Answer .....

(b) At time $t = 0$ the force $\mathbf{F}_2$ is removed. Find the acceleration of P and its magnitude. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Show that the velocity of P is never parallel to its acceleration. [3 marks]

Answer .....

(d) Find the speed of P at $t = 2$ s and the bearing on which it is then moving. [3 marks]

Answer .....

Total for D5: 11 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D6 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A particle P of mass 2.5 kg moves on a smooth horizontal surface under a single constant force $(5\mathbf{i} + \lambda\mathbf{j})$ N, where λ is a constant. At time $t = 0$, P passes through the origin O with velocity $(\mathbf{i} + 6\mathbf{j})\ \mathrm{m\ s^{-1}}$. The acceleration of P acts in the direction of $2\mathbf{i} - 3\mathbf{j}$.

(a) Find the value of λ. [2 marks]

Answer .....

(b) Find the time when P is due east of O, and the distance of P from O at that time. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the bearing on which P is moving at that time. [2 marks]

Answer .....

Total for D6: 8 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

No route is given. The hints are at the end of the sheet. Use one at a time.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E1 A particle of mass 3 kg moves under a constant force $\mathbf{F}$. At time $t = 0$ its velocity is $(-6\mathbf{i} + 8\mathbf{j})\ \mathrm{m\ s^{-1}}$ and at $t = 2$ s its velocity is $(-2\mathbf{i} + 6\mathbf{j})\ \mathrm{m\ s^{-1}}$. Find $\mathbf{F}$. Find the least speed of the particle and the time at which it happens. Find the distance of the particle from its position at $t = 0$ at that time. [11 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $\mathbf{a} = \dfrac{\mathbf{v} - \mathbf{u}}{t}$ and $\mathbf{F} = m\mathbf{a}$. Then write $\mathbf{v}$ at a general time t.

Hint 2 Write the speed squared, $|\mathbf{v}|^2$, as a quadratic in t. The speed is least when this quadratic is least.

Hint 3 Complete the square. Then use $\mathbf{r} = \mathbf{u}t + \dfrac{1}{2}\mathbf{a}t^2$ at that time and find its magnitude.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use F = ma with forces in i, j form to find an acceleration vector, its magnitude and its bearing.	○ ○ ○
I can find an unknown force, mass or component from a given acceleration.	○ ○ ○
I can use the constant acceleration formulae with vectors, and find speeds and distances as magnitudes.	○ ○ ○
I can find the acceleration and the force from positions or velocities at two times.	○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.