Vertical motion under gravity

122 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-vertical-motion-under-gravity/

Date

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A Do Now

Quick questions to start. They use what you need today.

A1 Needed today [3 marks]

A particle moves in a straight line with initial velocity $10\ \text{m s}^{-1}$ and constant acceleration $-4\ \text{m s}^{-2}$. Find the times when its displacement from the start is $8\ \text{m}$.

Answer .....

A2 Needed today [2 marks]

A car slows down uniformly from $12\ \text{m s}^{-1}$ to rest over a distance of $18\ \text{m}$. Find its deceleration.

Answer .....

A3 Common slip [2 marks]

A stone is dropped from rest. Kim finds its speed after $2.5\ \text{s}$ as $24.525\ \text{m s}^{-1}$. Find the speed using $g = 9.8\ \text{m s}^{-2}$ and give it to a suitable accuracy.

Answer .....

A4 Common slip [2 marks]

A ball is thrown upwards and lands on the ground $12\ \text{m}$ below the point where it was thrown. Taking upwards as positive, write down the acceleration a and the displacement s of the ball for the whole motion.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A stone is dropped from rest from a bridge $30\ \text{m}$ above a river. Find the time it takes to reach the water and its speed when it does.

Hint Take downwards as positive, so $a = 9.8$, $u = 0$ and $s = 30$. Use $s = ut + \frac{1}{2}at^2$ for t and $v^2 = u^2 + 2as$ for v.

B1 A ball is thrown vertically downwards with speed $3\ \text{m s}^{-1}$ from a window $22\ \text{m}$ above the ground. Find its speed when it hits the ground and the time it takes. [4 marks]

Answer .....

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EXAMPLE 2

A ball is projected vertically upwards from the ground with speed $19.6\ \text{m s}^{-1}$. Find the greatest height it reaches, the time it takes to reach this height, the total time it is in the air, and its velocity when it is $14.7\ \text{m}$ above the ground.

Hint Take upwards as positive, so $a = -9.8$. At the greatest height $v = 0$. It lands when $s = 0$ again. At a height there are two velocities: one on the way up and one on the way down.

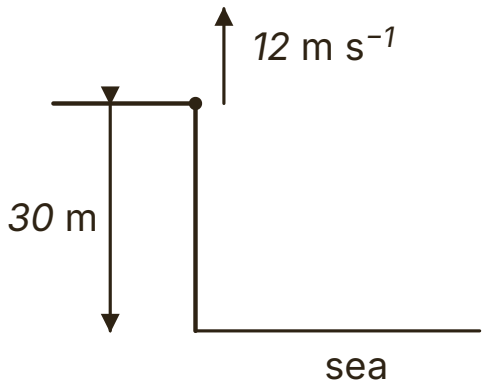
B2 A ball is projected vertically upwards from the ground with speed $24.5\ \text{m s}^{-1}$. Find the greatest height it reaches, the total time it is in the air, and its velocity $3\ \text{s}$ after it is projected. [5 marks]

Answer .....

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EXAMPLE 3

A stone is projected vertically upwards with speed $12\ \text{m s}^{-1}$ from the edge of a cliff $30\ \text{m}$ above the sea. Find the time it takes to reach the sea and its speed when it does.



Hint Take upwards as positive. The sea is $30\ \text{m}$ BELOW the start, so $s = -30$. The time is not twice the time to the top: that only brings the stone back to the edge of the cliff.

B3 A ball is projected vertically upwards with speed $9.8\ \text{m s}^{-1}$ from the top of a building $14.7\ \text{m}$ high. Find the time it takes to reach the ground, its speed when it does, and the greatest height of the ball above the ground. [6 marks]

Answer .....

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EXAMPLE 4

A ball is projected vertically upwards from the ground with speed $21\ \text{m s}^{-1}$. Find the length of time for which it is at least $18\ \text{m}$ above the ground.

Hint Find the two times when $s = 18$. The ball is at least $18\ \text{m}$ high between them.

B4 A ball is projected vertically upwards from the ground with speed $28\ \text{m s}^{-1}$. Find the length of time for which it is at least $30\ \text{m}$ above the ground, and its speed when it is $30\ \text{m}$ above the ground. [5 marks]

Answer .....

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EXAMPLE 5

A stone A is thrown vertically downwards with speed $2\ \mathrm{m\ s^{-1}}$ from the top of a building $50\ \mathrm{m}$ high. At the same instant a ball B is projected vertically upwards with speed $18\ \mathrm{m\ s^{-1}}$ from the ground directly below. Find when they are at the same height, and this height.

Hint Measure both heights from the ground, upwards positive. A starts at 50 with velocity -2. They are at the same height when the two height formulae are equal. The $-4.9t^2$ terms cancel.

B5 A ball P is projected vertically upwards from a point O with speed $14.7\ \mathrm{m\ s^{-1}}$. One second later a ball Q is projected vertically upwards from O with speed $19.6\ \mathrm{m\ s^{-1}}$. Find the time after P is projected when the balls collide, and their height above O then. [6 marks]

Answer .....

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C1 A ball is projected vertically upwards with speed $14\ \text{m s}^{-1}$ from a point $21\ \text{m}$ above the ground. Ben says: "It takes $\frac{14}{9.8} = 1.43$ s to reach the top, so it is in the air for $2 \times 1.43 = 2.86$ s." Explain Ben's mistake and find the correct time. [4 marks]

Answer .....

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C2 A hot-air balloon is rising vertically at a constant $4.9\ \text{m s}^{-1}$. When it is $29.4\ \text{m}$ above the ground a sandbag is released from it. Mia says the sandbag falls from rest, so it reaches the ground after $\sqrt{\dfrac{29.4}{4.9}} = 2.45$ s. Find the correct time and the speed of the sandbag when it reaches the ground. [5 marks]

Answer .....

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C3 A ball is projected vertically upwards with speed $7\ \mathrm{m\ s^{-1}}$ from a point $2.4\ \mathrm{m}$ above the ground and moves until it hits the ground. Ella says the ball travels $2.4\ \mathrm{m}$. Explain why she is wrong, find the total distance the ball travels, and find its speed when it hits the ground. [5 marks]

Answer .....

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C4 A ball is projected vertically upwards from the ground with speed $15\ \mathrm{m\ s^{-1}}$. Using a model in which the ball is a particle moving freely under gravity, find the greatest height it reaches. State, with a reason, whether the true greatest height is more or less than this. Give one other assumption of the model and one way to refine it. [5 marks]

Answer .....

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C5 A stone is projected vertically upwards from the ground. $3\ \text{s}$ later it is moving downwards with speed $7.7\ \text{m s}^{-1}$. Zac writes $7.7 = u - 9.8 \times 3$, and gets a greatest height of $70.2\ \text{m}$. Explain his mistake and find the correct greatest height. [4 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 A small ball is released from rest at a height of $4.9\ \text{m}$ above horizontal ground. It hits the ground and rebounds vertically upwards with $\frac{3}{5}$ of the speed it had just before it hit the ground. The ball is modelled as a particle moving freely under gravity, and the time it is in contact with the ground is ignored.

(a) Find the speed of the ball just before it hits the ground. [2 marks]

Answer .....

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(b) Find the greatest height the ball reaches after it rebounds. [3 marks]

Answer .....

(c) Find the time from the release of the ball until it hits the ground for the second time. [3 marks]

Answer .....

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(d) Taking downwards as positive, sketch a velocity–time graph for the motion of the ball from its release until it hits the ground for the second time. Mark the velocities at the start and the end of each straight line. [3 marks]

Answer .....

(e) Give one limitation of the model that would affect your answer to part (c). [2 marks]

Total for D1: 13 marks

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D2

A ball is projected vertically upwards with speed $9.8\ \text{m s}^{-1}$ from the top of a tower $24.5\ \text{m}$ high, and falls to the ground at the foot of the tower. Ria finds the time to reach the ground, taking upwards as positive: "$24.5 = 9.8t - 4.9t^2$, so $t^2 - 2t + 5 = 0$. The discriminant is $4 - 20 < 0$, so the ball never reaches the ground."

Ria is wrong.

(a) Explain Ria's mistake. [2 marks]

(b) Find the time taken for the ball to reach the ground. [3 marks]

Answer .....

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(c) Find the speed of the ball when it reaches the ground. [2 marks]

Answer .....

(d) Find the length of time for which the ball is above the top of the tower. [2 marks]

Answer .....

(e) Find the greatest height of the ball above the ground. [2 marks]

Answer .....

Total for D2: 11 marks

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D3 A stone P is released from rest at the top of a tower. One second later a stone Q is thrown vertically downwards from the same point. Both stones hit the ground at the same instant, $3\ \text{s}$ after P was released. Modelling the stones as particles moving freely under gravity, find the height of the tower, the speed with which Q was thrown, and the speed of each stone when it hits the ground. [8 marks]

Answer .....

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D4 A stone is released from rest at a point O near the top of a tall building and falls vertically past a window. The window is $2.4\ \text{m}$ tall and the stone takes $0.3\ \text{s}$ to fall from the top of the window to the bottom. The bottom of the window is $9.6\ \text{m}$ above horizontal ground. The stone is modelled as a particle moving freely under gravity.

(a) Show that the speed of the stone as it passes the top of the window is $6.53\ \text{m s}^{-1}$. [3 marks]

Answer .....

(b) Hence find the height of O above the top of the window. [2 marks]

Answer .....

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(c) Find the time from the release of the stone until it hits the ground. [3 marks]

Answer .....

(d) Taking downwards as positive, sketch a velocity–time graph for the motion of the stone from its release until it hits the ground. Mark the times at which it passes the top and the bottom of the window, and explain how the height of the window can be found from your graph. [3 marks]

Answer .....

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(e) In fact air resistance acts on the stone. Assuming the stone still passes the top of the window at $6.53\ \text{m s}^{-1}$, state, with a reason, whether the true height of O above the window is more or less than your answer to part (b). [2 marks]

Total for D4: 13 marks

D5 A ball P is projected vertically upwards from a point O with speed $24.5\ \text{m s}^{-1}$. One second later a ball Q is projected vertically upwards from O. The balls collide at the instant when P is at its greatest height.

(a) Find the greatest height of P above O and the time at which P reaches it. [3 marks]

Answer .....

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(b) Find the speed with which Q was projected. [3 marks]

Answer .....

(c) Find the velocity of Q at the instant of the collision. [2 marks]

Answer .....

(d) Find the greatest height above O that Q would have reached if the balls had not collided. [2 marks]

Answer .....

Total for D5: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A ball is projected vertically upwards from a point O. Points A and B are vertically above O, with B above A. The ball is above A for $3\ \text{s}$ and above B for $1\ \text{s}$. Show that AB does not depend on the speed of projection, and find AB. Given also that the greatest height of the ball above O is $19.6\ \text{m}$, find OA. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 The ball goes up and comes down in the same way. If it is above a point for T seconds, how long does it take to fall from the top back down to that point?

Hint 2 Let H be the greatest height. Falling from rest for $\frac{T}{2}$ seconds, the ball drops $\frac{1}{2}g\left(\frac{T}{2}\right)^2$. So $H - h = \frac{1}{8}gT^2$.

Hint 3 Write this for A ($T = 3$) and for B ($T = 1$) and subtract: H cancels. For OA, use $H = 19.6$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can choose a positive direction and give s, u, v and a the right signs, including below the start. ○ ○ ○

I can find the greatest height, the time of flight and the time for which an object is above a given height. ○ ○ ○

I can model an object released from a moving balloon, a bounce, or two objects in the air at once. ○ ○ ○

I can state the modelling assumptions and say how air resistance changes the answers. ○ ○ ○

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