Working with vectors

125 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-working-with-vectors/

Date

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A Do Now

Quick questions to start. Some use skills from GCSE and Pure Year 1; some set slips that cost marks.

A1 Needed today [2 marks]

Find the magnitude of $7\mathbf{i} - 24\mathbf{j}$ and a unit vector in its direction.

Answer .....

A2 Needed today [3 marks]

A point B is 8 km from A on a bearing of $150°$. Find how far east and how far south of A the point B is.

Answer .....

A3 Common slip [2 marks]

Write $45\ \text{km h}^{-1}$ in m s^{-1}.

Answer .....

A4 Common slip [2 marks]

A particle moves along a straight line. The positive direction is to the right. Its velocity is $-6\ \text{m s}^{-1}$. Ben says that its speed is $-6\ \text{m s}^{-1}$. Correct him, and state the direction in which the particle moves.

Answer .....

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A5 Common slip [3 marks]

Kai says that the angle between $-3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{i}$ is $53.1°$. Find the correct angle.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

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EXAMPLE 1

A cyclist rides along a straight road. East is the positive direction. She starts at O, rides 400 m east at $5\ \text{m s}^{-1}$, then turns and rides at $4\ \text{m s}^{-1}$ to a point 150 m west of O.

(a) Find the time for each part of the ride.

(b) Find the total distance travelled and the final displacement from O.

(c) Find the average speed and the average velocity for the whole ride.

Hint Distance and speed are scalars; displacement and velocity are vectors and keep their sign.

B1 [6 marks]

A runner runs along a straight path. North is the positive direction. She starts at O, runs 1200 m north at $8\ \text{m s}^{-1}$, then turns and runs back at $6\ \text{m s}^{-1}$ until she is 600 m north of O.

(a) Write down her velocity on the way back.

(b) Find the total distance she runs and her final displacement from O.

(c) Find her average speed and her average velocity.

Answer .....

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EXAMPLE 2

A particle moves with velocity $(-5\mathbf{i} + 12\mathbf{j})\ \mathrm{m\ s^{-1}}$, where $\mathbf{i}$ is east and $\mathbf{j}$ is north. Find its speed, the angle between its direction of motion and $\mathbf{i}$, and the bearing on which it moves.

Hint Always sketch the vector first. The angle with i is measured from the positive i direction; a bearing is measured clockwise from north.

B2 A particle moves with velocity $(6\mathbf{i} - 8\mathbf{j})\ \mathrm{m\ s^{-1}}$, where $\mathbf{i}$ is east and $\mathbf{j}$ is north. Find its speed, the angle between its direction of motion and $\mathbf{i}$, and the bearing on which it moves. [4 marks]

Answer .....

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EXAMPLE 3

(a) A ship sails at $15\ \text{km h}^{-1}$ on a bearing of $210°$. Write its velocity in terms of $\mathbf{i}$ (east) and $\mathbf{j}$ (north).

(b) A particle has velocity $(p\mathbf{i} + 4\mathbf{j})\ \text{m s}^{-1}$ and speed $5\ \text{m s}^{-1}$. Find the possible values of p.

(c) A particle moves parallel to $4\mathbf{i} - 3\mathbf{j}$ with speed $10\ \text{m s}^{-1}$. Find its velocity.

Hint A direction vector fixes the direction only: scale it to the speed.

B3 [5 marks]

(a) A boat moves at $20\ \text{km h}^{-1}$ on a bearing of $300°$. Write its velocity in terms of $\mathbf{i}$ (east) and $\mathbf{j}$ (north).

(b) A particle has velocity $(3\mathbf{i} + q\mathbf{j})\ \text{m s}^{-1}$ and speed $3\sqrt{5}\ \text{m s}^{-1}$. It moves on a bearing between $090°$ and $180°$. Find q.

(c) A particle moves parallel to $5\mathbf{i} + 12\mathbf{j}$ with speed $26\ \text{m s}^{-1}$. Find its velocity.

Answer .....

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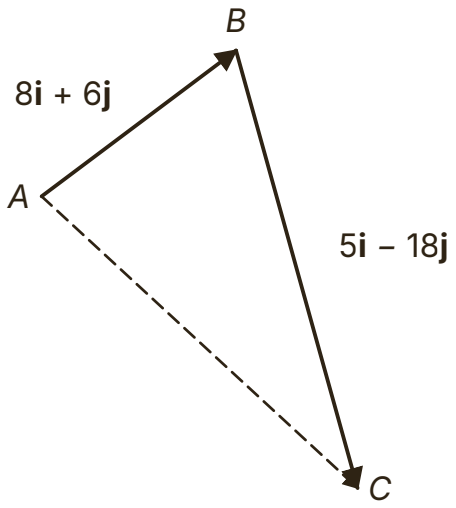
EXAMPLE 4

A drone flies from A to B, where $\overrightarrow{AB} = (8\mathbf{i} + 6\mathbf{j})$ m, and then from B to C, where $\overrightarrow{BC} = (5\mathbf{i} - 18\mathbf{j})$ m.

(a) Find $\overrightarrow{AC}$ and the distance AC.

(b) Find the total distance the drone flies.

(c) Find the angle between $\overrightarrow{AC}$ and $\mathbf{i}$.



Hint Add the displacements to find where it ends; add the lengths to find how far it flew.

B4 A hiker walks from A to B, where $\overrightarrow{AB} = (-9\mathbf{i} + 12\mathbf{j})$ km, and then from B to C, where $\overrightarrow{BC} = (15\mathbf{i} + 8\mathbf{j})$ km. Here $\mathbf{i}$ is east and $\mathbf{j}$ is north. [6 marks]

(a) Find $\overrightarrow{AC}$ and the distance AC.

(b) Find the total distance walked.

(c) Find the bearing of C from A.

Answer .....

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EXAMPLE 5

A particle moves with constant velocity. At $t = 0$ it is at the point with position vector $(2\mathbf{i} - 3\mathbf{j})$ m, and at $t = 3$ s it is at $(14\mathbf{i} + 6\mathbf{j})$ m.

(a) Find its velocity and speed.

(b) Find the time when it is due east of O, and its position then.

Hint r = r₀ + vt. "Due east of O" means the j-part is zero AND the i-part is positive.

B5 [6 marks]

A ship moves with constant velocity. At noon it is at $(-8\mathbf{i} + 5\mathbf{j})$ km and at 3 pm it is at $(4\mathbf{i} - 4\mathbf{j})$ km, where $\mathbf{i}$ is east and $\mathbf{j}$ is north, relative to a port O.

(a) Find its velocity and speed.

(b) Find the time when it is due south of O, and its distance from O then.

(c) Show that the ship is never due east of O.

Answer .....

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C1 A boat sails $(24\mathbf{i} + 7\mathbf{j})$ km and then $(-12\mathbf{i} + 9\mathbf{j})$ km. Sam says: "The boat has travelled $25 + 15 = 40$ km, so it is now 40 km from where it started." Explain Sam's mistake and find the distance of the boat from its start. [4 marks]

Answer .....

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C2 A particle moves with velocity $(-4\mathbf{i} - 3\mathbf{j})\ \text{m s}^{-1}$, where $\mathbf{i}$ is east and $\mathbf{j}$ is north. Ana says: "$\tan^{-1}\frac{3}{4} = 36.9°$, so the angle with $\mathbf{i}$ is $36.9°$ and the bearing is $036.9°$." Find the correct angle with $\mathbf{i}$ and the correct bearing. [4 marks]

Answer .....

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C3 A particle starts at the point with position vector $(2\mathbf{i} + \mathbf{j})$ m. It moves with constant velocity in the direction of $3\mathbf{i} - 4\mathbf{j}$ with speed $15\ \text{m s}^{-1}$. Jo says that its velocity is $(3\mathbf{i} - 4\mathbf{j})\ \text{m s}^{-1}$. Explain why Jo is wrong, and find the position vector of the particle after 4 s. [5 marks]

Answer .....

C4 A particle moves along a straight line with constant acceleration. Its velocity changes from $6\ \text{m s}^{-1}$ to $-4\ \text{m s}^{-1}$. Kai says: "Its speed fell by $10\ \text{m s}^{-1}$." Explain why Kai is wrong. [4 marks]

Answer .....

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C5 A boat moves at $12\ \text{km h}^{-1}$ on a bearing of $120°$, with $\mathbf{i}$ east and $\mathbf{j}$ north. Lee writes the velocity as $(12\cos 120°)\mathbf{i} + (12\sin 120°)\mathbf{j} = (-6\mathbf{i} + 10.4\mathbf{j})\ \text{km h}^{-1}$. Find the correct velocity, and the bearing on which Lee's velocity points. [4 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 In this question $\mathbf{i}$ is east, $\mathbf{j}$ is north and distances are in km. Two ships P and Q move with constant velocities. At noon P is at $(2\mathbf{i} + 10\mathbf{j})$ and Q is at $(-6\mathbf{i} - 2\mathbf{j})$. The velocity of P is $(4\mathbf{i} - 2\mathbf{j})\ \text{km h}^{-1}$ and the velocity of Q is $(6\mathbf{i} + 2\mathbf{j})\ \text{km h}^{-1}$. At time t hours after noon the position vectors of P and Q are $\mathbf{p}$ and $\mathbf{q}$.

(a) Find $\mathbf{p}$ and $\mathbf{q}$ in terms of t. [2 marks]

Answer .....

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(b) Show that the ships do not collide. [3 marks]

Answer .....

(c) Find the distance between the ships at 2 pm. [3 marks]

Answer .....

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(d) Find the time when Q is due north of P, and the distance between them then. [3 marks]

Answer .....

Total for D1: 11 marks

D2

A ship moves with constant velocity $(-8\mathbf{i} + 15\mathbf{j})\ \text{km h}^{-1}$, where $\mathbf{i}$ is east and $\mathbf{j}$ is north, relative to a fixed point O. Mo says: "$\tan^{-1}\frac{15}{8} = 61.9°$, so the ship sails on a bearing of $061.9°$."

Mo is wrong.

(a) Find the speed of the ship. [2 marks]

Answer .....

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(b) Explain Mo's mistake and find the bearing of the ship's course. [3 marks]

Answer .....

(c) At 9 am the ship is at the point $(4\mathbf{i} + 2\mathbf{j})$ km. Find the time when it has sailed 51 km. [2 marks]

Answer .....

(d) Find the distance of the ship from O at that time. [2 marks]

Answer .....

Total for D2: 9 marks

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D3 A ball rolls across a smooth horizontal floor with constant velocity. At time $t = 0$ it is at the point with position vector $(-3\mathbf{i} + 4\mathbf{j})$ m relative to a fixed point O, and at $t = 2$ s it is at $(5\mathbf{i} - 2\mathbf{j})$ m. At $t = 0$ the ball is 5 m from O. Find the other time when the ball is 5 m from O. [7 marks]

Answer .....

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D4 A walker walks 3 km on a bearing of $040°$ and then 5 km on a bearing of $130°$. Take $\mathbf{i}$ east and $\mathbf{j}$ north.

(a) Write each displacement in terms of $\mathbf{i}$ and $\mathbf{j}$. [4 marks]

Answer .....

(b) Find the distance and the bearing of the walker's final position from the start. [4 marks]

Answer .....

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(c) Explain how the distance in (b) can be found without using components. [2 marks]

Total for D4: 10 marks

D5 A particle P moves with constant velocity. At $t = 0$ it is at A, with position vector $(\mathbf{i} - 2\mathbf{j})$ m, and at $t = 4$ s it is at B, with position vector $(13\mathbf{i} + 14\mathbf{j})$ m.

(a) Find the velocity and the speed of P. [3 marks]

Answer .....

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(b) Later P passes through the point D with position vector $(k\mathbf{i} + 26\mathbf{j})$ m. Find the time when this happens and the value of k. [3 marks]

Answer .....

(c) A second particle Q moves with constant velocity from the point $(25\mathbf{i} - 2\mathbf{j})$ m at $t = 0$, and reaches D at the same time as P. Find the velocity and the speed of Q. [4 marks]

Answer .....

Total for D5: 10 marks

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D6 A model boat starts at a fixed point O. It moves with velocity $(8\mathbf{i} + 6\mathbf{j})\ \text{m s}^{-1}$ for 20 s, then with velocity $(-5\mathbf{i} + 12\mathbf{j})\ \text{m s}^{-1}$ for 30 s.

(a) Find the position vector of the boat at the end of its journey. [3 marks]

Answer .....

(b) Find the total distance travelled by the boat, and its final distance from O. [3 marks]

Answer .....

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(c) Find the magnitude of the average velocity of the boat for the whole journey. [2 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A drone starts at a fixed point O. It flies with velocity $(5\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-1}$ for T_1 s and then with velocity $(-3\mathbf{i} + 4\mathbf{j})\ \text{m s}^{-1}$ for T_2 s. It ends due north of O, 260 m from O. Its battery allows at most 400 m of flight. Find T_1 and T_2, decide whether the drone can make this flight, and find its average velocity. [10 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the final position as $T_1(5\mathbf{i} + 2\mathbf{j}) + T_2(-3\mathbf{i} + 4\mathbf{j})$. Due north means the $\mathbf{i}$ part is zero.

Hint 2 The $\mathbf{j}$ part is 260. Solve the two equations for T_1 and T_2.

Hint 3 The distance flown is speed $\times$ time on each part, not 260 m. The average velocity uses the displacement and the total time $T_1 + T_2$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

Tell a vector from a scalar, and give a speed (never negative) from a velocity.	○ ○ ○
Turn a speed and a bearing into i, j form, and back again.	○ ○ ○
Find a resultant displacement, a distance travelled and an average velocity.	○ ○ ○
Use r = r_0 + vt for an object moving with constant velocity.	○ ○ ○

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