Centres of mass and tilting

115 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-year-2-centres-of-mass-and-tilting/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks.

A1 Needed today [2 marks]

A uniform rod AB has length $4\ \text{m}$ and mass $8\ \text{kg}$. It rests horizontally on supports at A and C, where $AC = 3\ \text{m}$. Find the reaction at C.

Answer .....

A2 Needed today [2 marks]

A light rod AB of length $3\ \text{m}$ has a particle of mass $2\ \text{kg}$ at A and a particle of mass $4\ \text{kg}$ at B. Find the distance from A of the point about which the total moment of the two weights is zero.

Answer .....

A3 Common slip [2 marks]

Ria says: "The weight of a rod always acts at its midpoint." Explain when Ria is right and when she is wrong.

Answer .....

A4 Needed today [2 marks]

A beam of weight $150\ \text{N}$ carries a load of $90\ \text{N}$ and rests on two supports. One reaction is twice the other. Find the smaller reaction.

Answer .....

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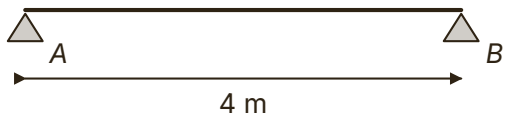
B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

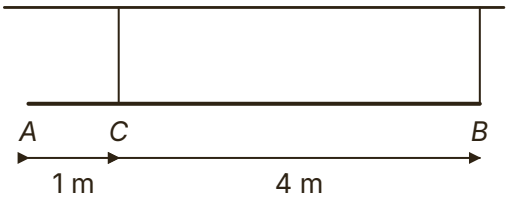
EXAMPLE 1

A non-uniform rod AB has length $4\ \text{m}$ and mass $12\ \text{kg}$. It rests horizontally on two supports, one at A and one at B. The reaction at B is $44.1\ \text{N}$. Find the distance of the centre of mass of the rod from A, and the reaction at A.



Hint Let the centre of mass G be $x\ \text{m}$ from A. The weight $12g$ acts at G, not at the midpoint. Take moments about A so that the unknown reaction at A has no moment.

B1 A non-uniform plank AB has length $5\ \text{m}$ and mass $20\ \text{kg}$. It hangs horizontally from two vertical strings attached at C and B, where $AC = 1\ \text{m}$. The tension in the string at C is $117.6\ \text{N}$. Find the distance of the centre of mass from A and the tension in the string at B. [5 marks]



Answer .....

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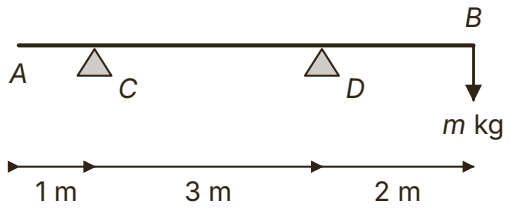
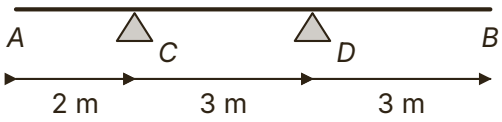
EXAMPLE 2

A uniform plank AB has length $6\ \text{m}$ and mass $30\ \text{kg}$. It rests horizontally on supports at C and D, where $AC = 1\ \text{m}$ and $AD = 4\ \text{m}$. (a) A particle of mass $m\ \text{kg}$ is placed at B and the plank is about to tilt about D. Find m and the reaction at D. (b) The particle is removed. Find the greatest mass that can be placed at A without the plank tilting.



Hint When the plank is about to tilt about D, it is about to lose contact with C, so the reaction at C is zero. Then take moments about D.

B2 A uniform beam AB has length $8\ \text{m}$ and mass $40\ \text{kg}$. It rests horizontally on supports at C and D, where $AC = 2\ \text{m}$ and $AD = 5\ \text{m}$. Find the greatest mass that can be placed (a) at B, (b) at A, without the beam tilting. [5 marks]



Answer .....

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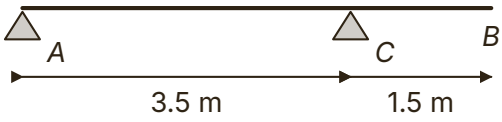
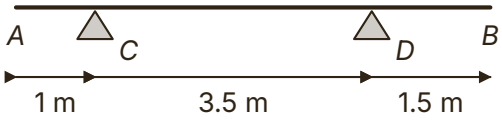
EXAMPLE 3

A uniform plank AB has length $5\ \text{m}$ and mass $25\ \text{kg}$. It rests horizontally on supports at A and C, where $AC = 3.5\ \text{m}$. A man of mass $70\ \text{kg}$ walks along the plank from A towards B. The man is modelled as a particle. Find how far from A he can walk before the plank tilts. Can he reach B?



Hint While he is between A and C the plank cannot tilt. Past C, it would tilt about C, lifting off A: set $R_A = 0$ with the man $x\ \text{m}$ from A.

B3 A uniform plank AB has length $6\ \text{m}$ and mass $40\ \text{kg}$. It rests horizontally on supports at C and D, where $AC = 1\ \text{m}$ and $AD = 4.5\ \text{m}$. A man of mass $75\ \text{kg}$ stands on the plank. (a) Find the greatest distance from A at which he can stand without the plank tilting. (b) Show that he can stand at A . [6 marks]



Answer .....

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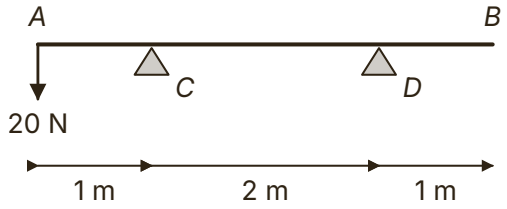
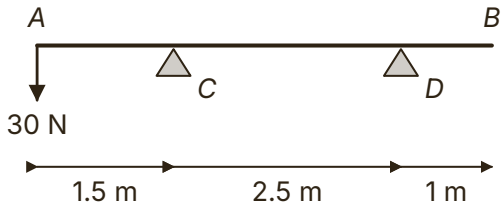
EXAMPLE 4

A non-uniform rod AB has length $4\ \text{m}$ and weight $W\ \text{N}$. Its centre of mass is at G, where $AG = x\ \text{m}$. The rod rests horizontally on supports at C and D, where $AC = 1\ \text{m}$ and $AD = 3\ \text{m}$. When a particle of weight $20\ \text{N}$ is placed at A, the rod is about to tilt about C. When this particle is removed and a particle of weight $30\ \text{N}$ is placed at B, the rod is about to tilt about D. Find W and x.



Hint Write one equation for each situation. In the first, $R_D = 0$; take moments about C. In the second, $R_C = 0$; take moments about D.

B4 A non-uniform rod AB has length $5\ \text{m}$ and weight $W\ \text{N}$. Its centre of mass is at G, where $AG = x\ \text{m}$. It rests horizontally on supports at C and D, where $AC = 1.5\ \text{m}$ and $AD = 4\ \text{m}$. When a particle of weight $30\ \text{N}$ is placed at A, the rod is about to tilt about C. When the particle is moved to B, the rod is about to tilt about D. Find W and x. [6 marks]



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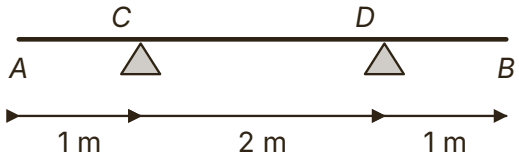
Answer .....

C Practice

A-Level

C1 A uniform plank AB has length $4\ \text{m}$ and mass $20\ \text{kg}$. It rests horizontally on supports at C and D, where $AC = 1\ \text{m}$ and $AD = 3\ \text{m}$. A load is placed at B. To find the greatest load before the plank tilts, Jo writes "$R_D = 0$" and takes moments about C. Explain Jo's mistake and find the greatest load. [5 marks]



Answer .....

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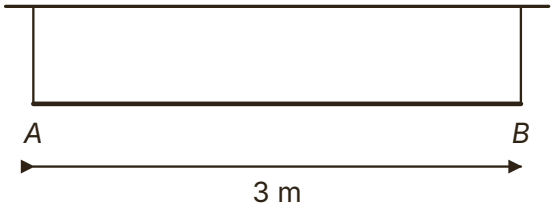
C2 A non-uniform rod AB has length $3\ \text{m}$ and mass $6\ \text{kg}$. It hangs horizontally from vertical strings at A and B. The tension at A is twice the tension at B. Kim says the tensions must be equal "because the weight acts at the middle". Explain Kim's mistake and find the distance of the centre of mass from A. [5 marks]



Answer .....

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C3 A non-uniform rod AB has length $6\ \text{m}$ and weight $W\ \text{N}$, with centre of mass G where $AG = x\ \text{m}$. It rests on supports at C and D, where $AC = 2\ \text{m}$ and $AD = 5\ \text{m}$. With a particle of weight $40\ \text{N}$ at A, the rod is about to tilt about C. With the same particle at B instead, the rod is about to tilt about D. Lee writes one equation: "$40 \times 2 + 40 \times 1 = W(x - 2) + W(5 - x)$". Explain Lee's mistake and find W and x. [6 marks]

Answer .....

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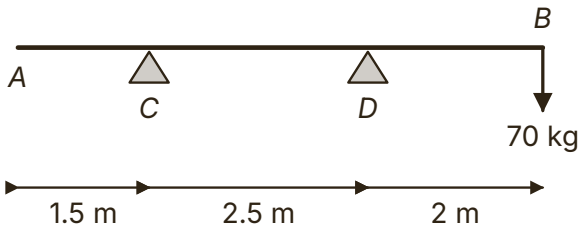
C4 A uniform plank AB has length $6\ \text{m}$ and mass $20\ \text{kg}$. It rests horizontally on supports at C and D, where $AC = 1.5\ \text{m}$ and $AD = 4\ \text{m}$. A man of mass $70\ \text{kg}$ stands at B. Ben works out the reactions and gets $R_D = 1350\ \text{N}$ and $R_C = -470\ \text{N}$. He says: "The support at C pulls the plank down with a force of $470\ \text{N}$." (a) Explain why Ben is wrong. (b) The support at D is moved towards B so that the man can stand at B without the plank tilting. Find the least possible distance AD. [5 marks]



Answer .....

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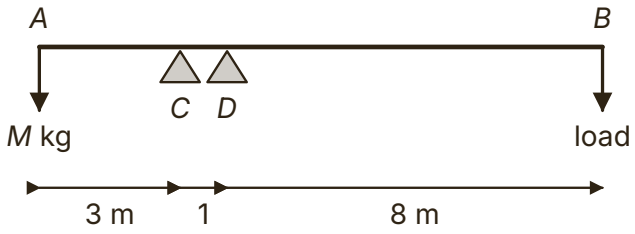
D1 A crane jib AB is modelled as a uniform rod of length $12\ \text{m}$ and mass $600\ \text{kg}$. It rests horizontally on two supports at the top of the tower, at C and D, where $AC = 3\ \text{m}$ and $AD = 4\ \text{m}$. A counterweight of mass $M\ \text{kg}$ is fixed at A and a load can be hung from B. The counterweight and the load are modelled as particles.

(a) With no load, find the range of values of M for which the jib does not tilt. [4 marks]



Answer .....

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(b) Given that $M = 600$, find the greatest load that can be hung from B. [3 marks]

Answer .....

(c) The counterweight is changed to $750\ \text{kg}$. Show that the jib now needs a load at B, and find the range of loads $L\ \text{kg}$ that can be hung from B. [4 marks]

Answer .....

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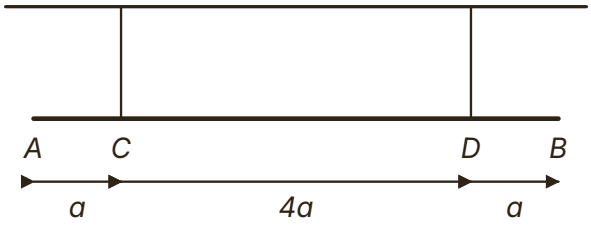
(d) State one way in which the model could be refined. [2 marks]

Total for D1: 13 marks

D2 A non-uniform beam AB has length $6a$ and weight W. Its centre of mass is G. The beam hangs horizontally from two vertical wires attached at C and D, where $AC = DB = a$.

(a) When a particle of weight W hangs from A, the beam is on the point of tipping. Show that $AG = 2a$. [3 marks]



Answer .....

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(b) The particle is moved from A to B. Find the tension in each wire, in terms of W. [3 marks]

Answer .....

(c) Find the greatest weight that can hang from B without the beam tipping. [3 marks]

Answer .....

Total for D2: 9 marks

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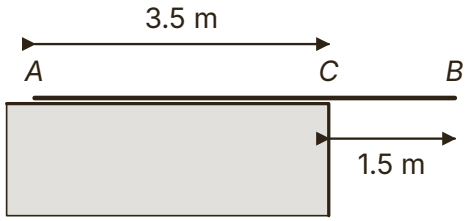
D3

A uniform plank AB has length $5\ \text{m}$ and mass $20\ \text{kg}$. It lies on a flat roof with $3.5\ \text{m}$ on the roof and $1.5\ \text{m}$ overhanging the edge C, as shown. Sam has mass $50\ \text{kg}$. He says: "Most of the plank is on the roof, so I can walk all the way to B without it tipping."

Sam is wrong.

(a) Find how far beyond C Sam can walk before the plank tips. [4 marks]



Answer .....

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(b) A bag of sand of mass m kg is placed on the plank at A. Find the least value of m that lets Sam stand at B. [4 marks]

Answer .....

Total for D3: 8 marks

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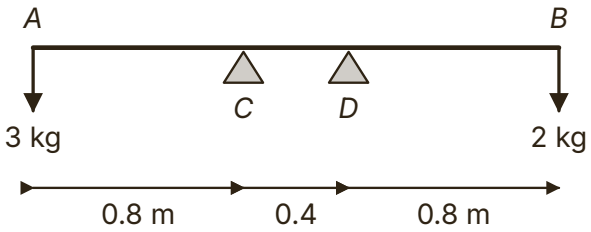
D4 A non-uniform rod AB has length $2\ \text{m}$ and mass $5\ \text{kg}$. Its centre of mass is G. The rod rests horizontally on supports at C and D, where $AC = 0.8\ \text{m}$ and $AD = 1.2\ \text{m}$. A particle of mass $3\ \text{kg}$ is attached at A and a particle of mass $2\ \text{kg}$ is attached at B. The rod is in equilibrium. Find the range of possible values of AG. [7 marks]



Answer .....

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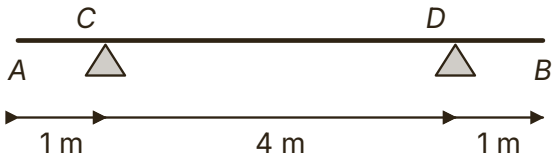
D5 A non-uniform plank AB has length $6\ \text{m}$ and mass $M\ \text{kg}$. Its centre of mass is G, where $AG = x\ \text{m}$. The plank rests horizontally on supports at C and D, where $AC = 1\ \text{m}$ and $AD = 5\ \text{m}$. A child of mass $25\ \text{kg}$ is modelled as a particle.

(a) When the child stands at A, the plank is about to tilt. Show that $M(x - 1) = 25$. [3 marks]



Answer .....

(b) When the child stands at B, the reaction at D is three times the reaction at C. Show that $Mx = 4M - 50$. [3 marks]

Answer .....

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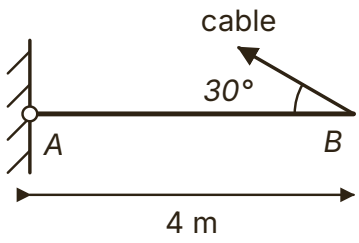
(c) Find M and x. [3 marks]

Answer .....

Total for D5: 9 marks

D6 A non-uniform beam AB has length $4\ \text{m}$ and mass $30\ \text{kg}$. The end A is freely hinged to a vertical wall. The beam is held horizontal by a light cable attached at B and to a point on the wall above A. The cable makes an angle of 30° with the beam. The tension in the cable is $200\ \text{N}$.

(a) Find the distance from A of the centre of mass of the beam. [4 marks]



Answer .....

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(b) A particle of mass $10\ \text{kg}$ is hung from B. The beam stays horizontal and the cable stays in the same position. Find the new tension. [3 marks]

Answer .....

(c) Explain why you do not need the force at the hinge to answer (a) and (b). [2 marks]

Total for D6: 9 marks

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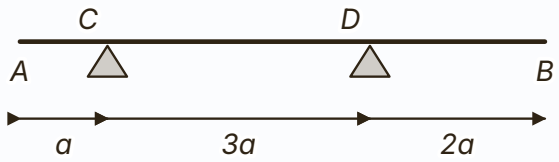
No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A uniform rod AB has length $6a$ and weight W. It rests horizontally on supports at C and D, where $AC = a$ and $DB = 2a$. A particle of weight kW, where $k > 0$, is placed on the rod at the point P, where $AP = x$. Find, in terms of a and k, the set of values of x for which the rod does not tilt. Hence find the greatest value of k for which the particle can be placed anywhere on the rod. [9 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 The rod does not tilt while both reactions are greater than or equal to zero.

Hint 2 Moments about C give R_D; moments about D give R_C. The centre of the rod is $2a$ from C and a from D.

Hint 3 "Anywhere on the rod" means the set must contain every x with $0 \leq x \leq 6a$. Each end gives one condition on k.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the position of the centre of mass of a non-uniform rod from the forces on it.	○ ○ ○
I can use the fact that the reaction at the other support is zero when a rod is about to tilt.	○ ○ ○
I can solve two-scenario problems with simultaneous equations.	○ ○ ○
I can find the range of loads or positions for which a rod stays in equilibrium.	○ ○ ○

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