A-LEVEL MATHS (EDEXCEL 9MA0) · MECHANICS YEAR 2 · CHAPTER 8: FURTHER KINEMATICS · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-mechanics-year-2-chapter-8-a-star/

Further kinematics: A* questions

96 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 A particle P moves along a straight line. At time t seconds, $0 \leqslant t \leqslant 2\pi$, the velocity of P is v m s^{-1}, where $v = 6\cos t + 8\sin t - 5$. When $t = 0$, P is at the fixed point O on the line.

(a) Express $6\cos t + 8\sin t$ in the form $R\cos(t - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. Give the value of α to 4 decimal places. [3 marks]

Answer .....

(b) Find the greatest speed of P, and the value of t at which it occurs. [3 marks]

Answer .....

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(c) Find the exact total distance travelled by P for $0 \leqslant t \leqslant 2\pi$. [6 marks]

Answer .....

Total for X1: 12 marks

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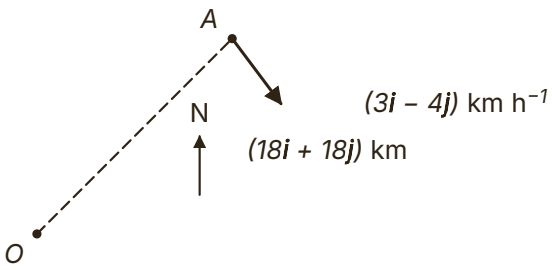
X2 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. At 12:00 a ship A is at the point with position vector $(18\mathbf{i} + 18\mathbf{j})$ km relative to a harbour O, and moves with constant velocity $(3\mathbf{i} - 4\mathbf{j})$ km h^{-1}. At 12:00 a patrol boat B leaves O. It moves in a straight line at a constant speed of 13 km h^{-1}, in the direction that makes it meet A. Find the time at which B meets A, the velocity of B, and the bearing on which B moves. [9 marks]



Answer Time ... , velocity ... , bearing ...

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X3 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle P is projected from a point O on horizontal ground and moves freely under gravity. When $t = 3$ seconds, the velocity of P is $(15\mathbf{i} - 9.8\mathbf{j})\ \mathrm{m\ s^{-1}}$.

(a) Find the velocity of projection of P, and the angle between the direction of projection and the horizontal. [3 marks]

Answer .....

(b) Find the length of time for which the speed of P is at most $17\ \mathrm{m\ s^{-1}}$. [5 marks]

Answer .....

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(c) P is at a height of h metres above the ground at two times that differ by 2 seconds. Find h. [4 marks]

Answer .....

(d) State one limitation of the model, and a refinement that would make it more realistic. [2 marks]

Total for X3: 14 marks

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X4 A particle P moves in a horizontal plane. At time t seconds, $t \geqslant 0$, the velocity of P is $\mathbf{v}\ \mathrm{m\ s^{-1}}$, where

$$\mathbf{v} = 10\mathrm{e}^{-t}(\cos t\,\mathbf{i} + \sin t\,\mathbf{j}).$$

When $t = 0$, P is at the fixed point O.

(a) Show that the speed of P is $10\mathrm{e}^{-t}\ \mathrm{m\ s^{-1}}$, and find the acceleration of P in terms of t. [3 marks]

Answer .....

(b) Show that the acceleration of P can be written as $10\sqrt{2}\,\mathrm{e}^{-t}\left(\cos(t+\frac{3\pi}{4})\mathbf{i} + \sin(t+\frac{3\pi}{4})\mathbf{j}\right)$. Hence find the angle between the direction of motion of P and its acceleration, and show that this angle is the same at every time. [4 marks]

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(c) Show that $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-t}(\sin t - \cos t)\right) = 2\mathrm{e}^{-t}\cos t$ and $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-t}(\sin t + \cos t)\right) = -2\mathrm{e}^{-t}\sin t$. Hence find the position vector of P at time t. [6 marks]

Answer .....

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(d) Find, in exact form, the position vector of P at the first time after $t = 0$ at which P is moving parallel to $\mathbf{j}$. Describe what happens to P as t becomes large. [4 marks]

Total for X4: 17 marks

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X5 A car P starts from rest at a point O on a straight horizontal road and moves along the road. At time t seconds after it starts, the acceleration of P is $5\mathrm{e}^{-\frac{t}{2}}\ \mathrm{m\ s^{-2}}$. At the instant P starts, a cyclist Q passes O moving in the same direction at a constant speed of $6\ \mathrm{m\ s^{-1}}$. The car and the cyclist are modelled as particles.

(a) Find the velocity of P and the displacement of P from O at time t seconds. [4 marks]

Answer .....

(b) Find, in exact form, the greatest distance between Q and P before P overtakes Q. [4 marks]

Answer .....

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(c) P overtakes Q when $t = T$. Show that $T = 5 - 5\mathrm{e}^{-\frac{T}{2}}$. [2 marks]

(d) Use the iteration $T_{n+1} = 5 - 5\mathrm{e}^{-\frac{T_n}{2}}$ with $T_0 = 4$ to find T_1, T_2 and T_3, giving each to 4 decimal places. [3 marks]

Answer .....

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(e) Show that $T = 4.46$ correct to 3 significant figures. Hence find the speed of P as it overtakes Q. [4 marks]

Total for X5: 17 marks

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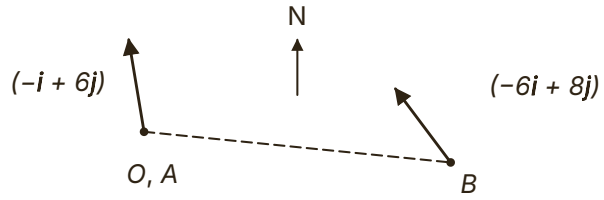
X6 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Distances are in kilometres. At 12:00 a ship A is at a harbour O, and a ship B is at the point with position vector $(20\mathbf{i} - 2\mathbf{j})$. B moves with constant velocity $(-6\mathbf{i} + 8\mathbf{j})\ \mathrm{km\ h^{-1}}$. A moves with velocity $(-\mathbf{i} + 6\mathbf{j})\ \mathrm{km\ h^{-1}}$ until 13:00. At 13:00 the velocity of A changes to $(-3\mathbf{i} + 4\mathbf{j})\ \mathrm{km\ h^{-1}}$, and it keeps this velocity. Find the least distance between the two ships between 12:00 and 16:00, the time at which it occurs, and the bearing of B from A at that time. [11 marks]



Answer Least distance ... at ... , bearing ...

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X7 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle P moves in a horizontal plane. At time t seconds, $t \geqslant 0$, the velocity of P is $\mathbf{v}\ \mathrm{m\,s^{-1}}$, where $\mathbf{v} = (3t^2 - 12t + c)\mathbf{i} + (2 - 4t)\mathbf{j}$ and c is a constant. When $t = 1$, P is moving on a bearing of 135°.

(a) Show that $c = 11$. [3 marks]

(b) Find the time at which the acceleration of P is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$, and the magnitude of the acceleration at that time. [5 marks]

Answer .....

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(c) When $t = 2$, P is at the point with position vector $(4\mathbf{i} - \mathbf{j})$ m relative to a fixed point O. Find the position vector of P at time t. [4 marks]

Answer .....

(d) Show that there is exactly one time $t \geqslant 0$ at which P is due east of O, and find the distance of P from O at that time. [4 marks]

Total for X7: 16 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can solve interception and least-distance problems with position vectors, including a velocity that changes part way.	○	○	○
I can use vectors with projectiles to set up and solve conditions on speed and height.	○	○	○
I can use harmonic form, exponentials and iteration with variable acceleration in a straight line.	○	○	○
I can differentiate and integrate vectors with trig and exponential terms, finding constants as vectors.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Write $v = 10\cos(t - \alpha) - 5$. What are the greatest and least values of v? Speed is the size of v.

Hint 2 P is at rest when $\cos(t - \alpha) = \frac{1}{2}$. There are two such times in the interval, so P moves in three stages.

Hint 3 For exact values, note that $6\sin t - 8\cos t = 10\sin(t - \alpha)$, so $x = 10\sin(t - \alpha) - 5t + 8$. Add the three stage distances.

X2

Hint 1 Write the position vector of A at time t.

Hint 2 B starts at O and moves $13t$ km in a straight line. They meet when the distance of A from O is $13t$.

Hint 3 Solve the quadratic and keep the positive root. The velocity of B is the meeting point divided by the time taken.

X3

Hint 1 Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ backwards, with $\mathbf{a} = -9.8\mathbf{j}$.

Hint 2 The horizontal component of the velocity is always 15. Write the speed squared and compare it with 17^2. Check that P has not landed.

Hint 3 The flight is symmetric about the time of the greatest height. Two times 2 seconds apart must sit 1 second either side of it.

X4

Hint 1 Differentiate each component with the product rule. For the speed use $\cos^2 t + \sin^2 t = 1$.

Hint 2 Expand $\cos(t + \frac{3\pi}{4})$ and $\sin(t + \frac{3\pi}{4})$ with the addition formulae. A vector $k(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$ with $k > 0$ makes an angle θ with $\mathbf{i}$.

Hint 3 The two derivatives give you the integrals of $\mathrm{e}^{-t}\cos t$ and $\mathrm{e}^{-t}\sin t$. The constant of integration is a vector: use $\mathbf{r} = \mathbf{0}$ when $t = 0$.

X5

Hint 1 Integrate twice. Each time, find the constant from the values when $t = 0$.

Hint 2 Q is ahead at first. The gap grows while Q is faster and shrinks once P is faster.

Hint 3 For the last part, use a sign change on $4T - 20 + 20\mathrm{e}^{-T/2}$ over the interval $[4.455, 4.465]$. Then use $\mathrm{e}^{-T/2} = 1 - \frac{T}{5}$ in v.

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X6

Hint 1 The velocity of A changes at 13:00, so write $\mathbf{r}_A$ twice: once for $0 \leqslant t \leqslant 1$ and once for $t \geqslant 1$, starting from where A is at 13:00.

Hint 2 Check whether the ships get closer or further apart before 13:00. Then work with $\overrightarrow{AB}$ for $t \geqslant 1$.

Hint 3 Write $\left|\overrightarrow{AB}\right|^2$ as a quadratic in t and complete the square. The bearing of B from A uses the direction of $\overrightarrow{AB}$.

X7

Hint 1 Draw the direction of a bearing of 135°. Which way do the $\mathbf{i}$ and $\mathbf{j}$ components of the velocity point, and how do their sizes compare?

Hint 2 Parallel to $\mathbf{i} - 2\mathbf{j}$ means $\mathbf{a} = \lambda(\mathbf{i} - 2\mathbf{j})$ for some λ. Find λ from the component you know.

Hint 3 The constant of integration is a vector found from the position at $t = 2$. Due east of O needs the $\mathbf{j}$ component zero and the $\mathbf{i}$ component positive.

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