Equilibrium of a rigid body

99 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-year-2-equilibrium-of-a-rigid-body/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks.

A1 Needed today [2 marks]

A light rod AB has length $5\ \text{m}$ and midpoint M. A force of $6\ \text{N}$ acts vertically upwards at A and a force of $4\ \text{N}$ acts vertically downwards at B. Find the resultant moment about M.

Answer .....

A2 Needed today [2 marks]

A uniform plank has mass $18\ \text{kg}$ and length $2.4\ \text{m}$. Find its weight and state where on the plank the weight acts.

Answer .....

A3 Needed today [2 marks]

A body of weight $90\ \text{N}$ is held at rest by two vertical upward forces. One of the forces is $34\ \text{N}$. Find the other.

Answer .....

A4 Common slip [2 marks]

A light rod AB of length $4\ \text{m}$ has a force of $5\ \text{N}$ acting vertically upwards at each end. Amir says: "The resultant moment about the midpoint is zero, so the rod is in equilibrium." Explain why Amir is wrong.

Answer .....

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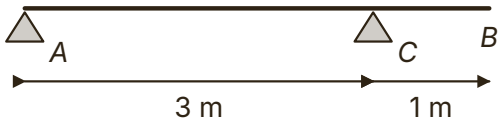
PART 1 Rods resting on two supports

EXAMPLE 1 · I DO

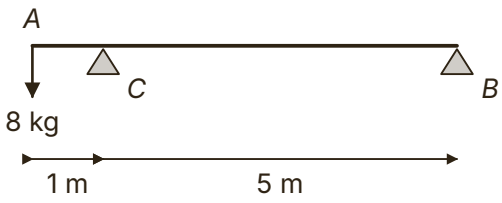
A uniform rod AB has length $4\ \text{m}$ and mass $10\ \text{kg}$. It rests horizontally on two supports, at A and at C, where $AC = 3\ \text{m}$. Find the reaction at each support.



Hint Draw the forces: R_A and R_C upwards, the weight $10g$ at the midpoint. Take moments about A: then R_A has no moment and only R_C is unknown.

YOUR TURN · WE DO

B1 A uniform beam AB has length $6\ \text{m}$ and mass $20\ \text{kg}$. It rests horizontally on supports at C and at B, where $AC = 1\ \text{m}$. A particle of mass $8\ \text{kg}$ is placed on the beam at A. Find the reaction at each support. [5 marks]



Answer .....

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C1 Dev says: "A uniform rod resting horizontally on two supports always has equal reactions at the supports." A uniform rod AB has length $6\ \mathrm{m}$ and mass $10\ \mathrm{kg}$. It rests on supports at A and C, where $AC = 4\ \mathrm{m}$. Find both reactions, and state when Dev would be right. [4 marks]

Answer .....

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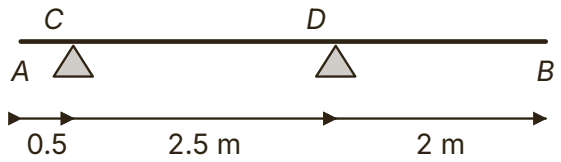
C2 A uniform beam AB has length $5\ \text{m}$ and mass $40\ \text{kg}$. It rests horizontally on supports at C and D, where $AC = 0.5\ \text{m}$ and $AD = 3\ \text{m}$. Zara takes moments about C and writes "$R_D \times 3 = 40g \times 2.5$". Explain Zara's mistake and find the reaction at each support. [5 marks]



Answer .....

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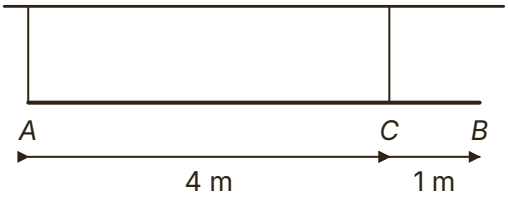
PART 2 Strings and unknown weights

EXAMPLE 2 · I DO

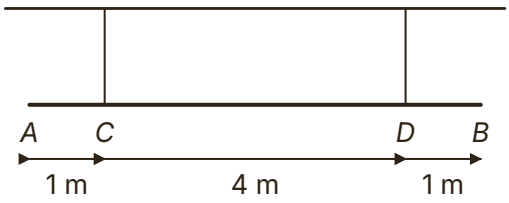
A uniform rod AB has length $5\ \text{m}$ and mass $12\ \text{kg}$. It hangs horizontally from two vertical strings, attached at A and at C, where $AC = 4\ \text{m}$. A particle of mass $3\ \text{kg}$ is attached to the rod at the point P. The tension in the string at C is twice the tension in the string at A. Find the distance AP.



Hint Call the tension at A T, so the tension at C is $2T$. Resolving gives T. Then take moments about A with $AP = x$.

YOUR TURN · WE DO

B2 A uniform rod AB has length $6\ \text{m}$ and mass $10\ \text{kg}$. It hangs horizontally from two vertical strings attached at C and D, where $AC = 1\ \text{m}$ and $AD = 5\ \text{m}$. A particle of mass $5\ \text{kg}$ is attached to the rod at P. The tension in the string at D is 1.5 times the tension in the string at C. Find the distance AP. [5 marks]



Answer .....

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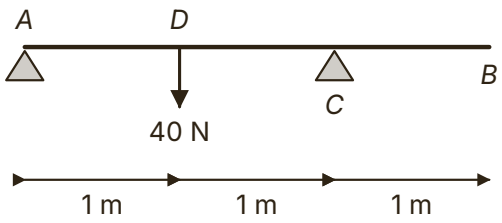
EXAMPLE 3 · I DO

A uniform rod AB has length $3\ \text{m}$ and weight $W\ \text{N}$. It rests horizontally on supports at A and C, where $AC = 2\ \text{m}$. A particle of weight $40\ \text{N}$ is placed on the rod at D, where $AD = 1\ \text{m}$. (a) Show that the reaction at C is $\dfrac{3W + 80}{4}\ \text{N}$. (b) The reaction at C is twice the reaction at A. Find W.



Hint For (a), take moments about A: the weight W acts $1.5\ \text{m}$ from A. For (b), find R_A in terms of W by resolving, then use $R_C = 2R_A$.

YOUR TURN · WE DO

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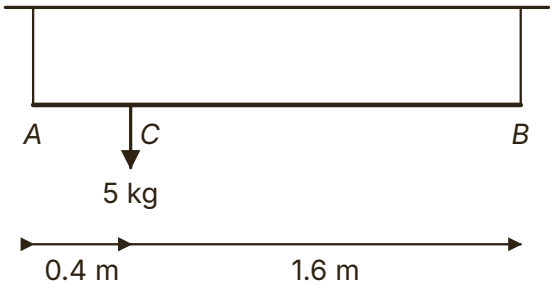
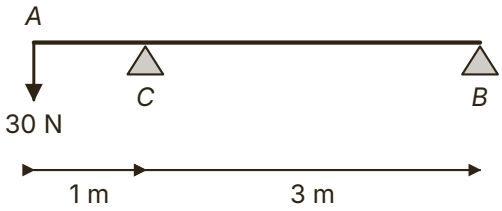
PRACTICE · YOU DO

C3 A uniform rod AB has length $2\ \text{m}$ and mass $3\ \text{kg}$. It hangs horizontally from vertical strings at A and B. A particle of mass $5\ \text{kg}$ is attached at C, where $AC = 0.4\ \text{m}$. Ben takes moments about A and writes "$T_B \times 2 = 5 \times 0.4$". Explain the two mistakes Ben has made, and find the tension in each string. [5 marks]



B3 A uniform rod AB has length $4\ \text{m}$ and weight $W\ \text{N}$. It rests horizontally on supports at C and B, where $AC = 1\ \text{m}$. A particle of weight $30\ \text{N}$ hangs from the rod at A. (a) Show that the reaction at B is $\dfrac{W - 30}{3}\ \text{N}$. (b) The reaction at C is 4 times the reaction at B. Find W. [6 marks]



Answer .....

Answer .....

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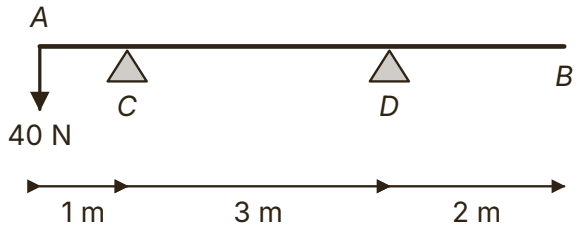
C4 A uniform rod AB has length $6\ \text{m}$ and weight $W\ \text{N}$. It rests horizontally on supports at C and D, where $AC = 1\ \text{m}$ and $AD = 4\ \text{m}$. A particle of weight $40\ \text{N}$ is placed on the rod at A. The reaction at C is $70\ \text{N}$. Find W and the reaction at D. [5 marks]



Answer .....

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PART 3 Hinged rods and rods at an angle

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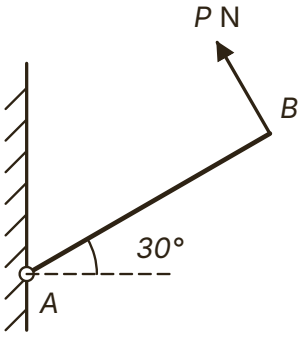
EXAMPLE 4 · I DO

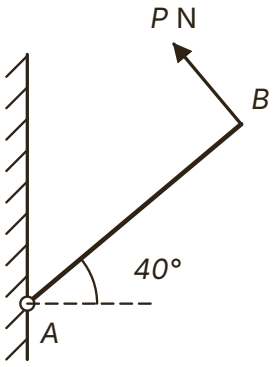
A uniform rod AB has length $3\ \text{m}$ and mass $5\ \text{kg}$. The end A is freely hinged to a vertical wall. The rod rests in equilibrium at 30° above the horizontal, held by a force $P\ \text{N}$ at B. (a) The force acts perpendicular to AB, as shown. Find P. (b) The force is now horizontal, towards the wall, and the rod rests at the same angle. Find the new value of P.



Hint Take moments about the hinge A, so the unknown force at the hinge has no moment. The weight is vertical: its perpendicular distance from A is $1.5\cos 30^\circ$. A horizontal force at B has perpendicular distance $3\sin 30^\circ$ from A.

YOUR TURN · WE DO

B4 A uniform rod AB, of length $4\ \text{m}$ and mass $6\ \text{kg}$, is hinged to a wall at A and rests at 40° above the horizontal, held by a force $P\ \text{N}$ at B. Find P when the force is (a) perpendicular to AB, as shown, (b) horizontal. [6 marks]



Answer .....

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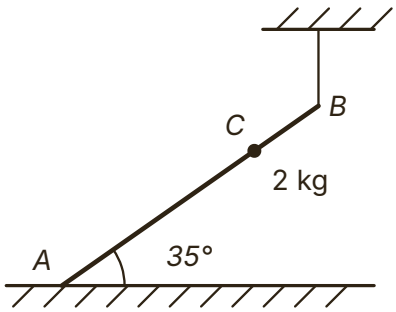
EXAMPLE 5 · I DO

A uniform rod AB has length $4\ \text{m}$ and mass $8\ \text{kg}$. The end A rests on smooth horizontal ground. The rod is held in equilibrium at an angle of 35° to the ground by a light vertical string attached at B. A particle of mass $2\ \text{kg}$ is attached to the rod at C, where $AC = 3\ \text{m}$. Find the tension in the string and the reaction at A.



Hint The ground is smooth, so the reaction at A is vertical, and every force on the rod is vertical. Take moments about A: the perpendicular distance from A to a vertical line of action is the distance along the rod times $\cos 35^\circ$.

YOUR TURN · WE DO

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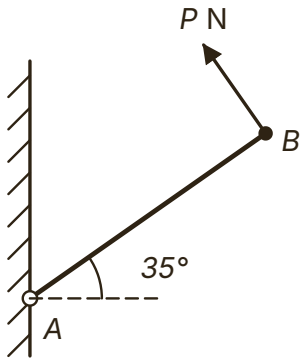
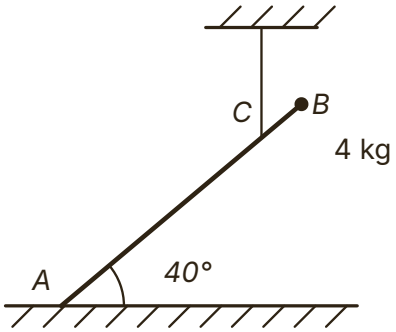
PRACTICE · YOU DO

C5 A uniform rod AB has length $2.4\ \text{m}$ and mass $4\ \text{kg}$. The end A is freely hinged to a vertical wall. A particle of mass $1.5\ \text{kg}$ is attached to the rod at B. The rod rests in equilibrium at 35° above the horizontal, held by a force P N at B. (a) The force acts perpendicular to AB, as shown. Find P. (b) The force is now vertical and the rod rests at the same angle. Find the new value of P. [5 marks]



B5 A uniform rod AB has length $6\ \text{m}$ and mass $10\ \text{kg}$. The end A rests on smooth horizontal ground and the rod is held at 40° to the ground by a light vertical string attached at C, where $AC = 5\ \text{m}$. A particle of mass $4\ \text{kg}$ is attached at B. Find the tension in the string and the reaction at A. [6 marks]



Answer .....

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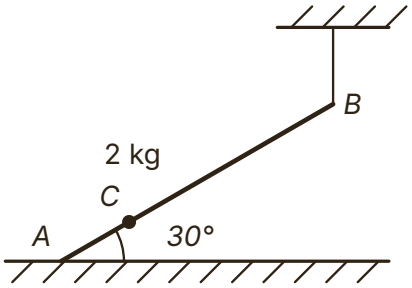
C6 A uniform rod AB has length $4\ \text{m}$ and mass $5\ \text{kg}$. The end A rests on smooth horizontal ground and the rod is held at 30° to the ground by a light vertical string attached at B. A particle of mass $2\ \text{kg}$ is attached at C, where $AC = 1\ \text{m}$. Ola takes moments about A and writes "$T \times 4 = 5g \times 2\cos 30^\circ + 2g \times 1 \times \cos 30^\circ$". Explain Ola's mistake, and find the tension in the string and the reaction at A. [5 marks]

Answer .....



Answer .....

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C7 A uniform rod AB has length $5\ \text{m}$ and mass $12\ \text{kg}$. The end A rests on smooth horizontal ground. A particle of mass $m\ \text{kg}$ is attached to the rod at B. The rod is held in equilibrium at 25° to the ground by a light vertical string attached to the rod at C. The reaction at A is $29.4\ \text{N}$ and the tension in the string is $147\ \text{N}$. Find m and the distance AC, and explain why the angle 25° is not needed. [7 marks]

Answer .....

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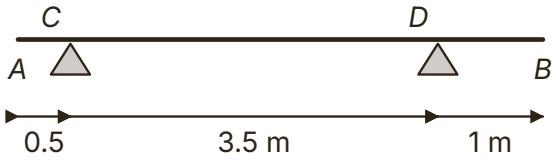
D1 A uniform plank AB has length $5\ \text{m}$ and mass $25\ \text{kg}$. It rests horizontally on two supports at C and D, where $AC = 0.5\ \text{m}$ and $AD = 4\ \text{m}$. A child of mass $40\ \text{kg}$ stands on the plank at the point E.

(a) $AE = 4.5\ \text{m}$. Find the reaction at each support. [4 marks]



Answer .....

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(b) The child moves to a new position on the plank, between C and D. The reaction at D is now twice the reaction at C. Find the new distance AE. [4 marks]

Answer .....

(c) State one modelling assumption you have made about the child, and explain how it was used. [2 marks]

Total for D1: 10 marks

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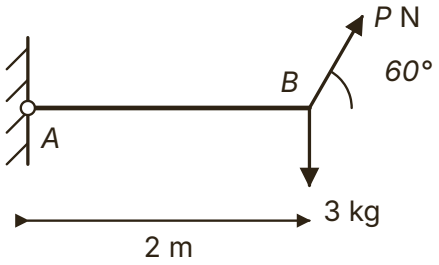
D2

A uniform rod AB has length $2\ \text{m}$ and mass $5\ \text{kg}$. The end A is freely hinged to a vertical wall and the rod rests horizontally. A particle of mass $3\ \text{kg}$ is attached at B. The rod is held by a force $P\ \text{N}$ acting at B, at an angle of 60° above the rod, as shown. Eve takes moments about A and writes "$P \times 2\cos 60^\circ = 5g \times 1 + 3g \times 2$".

Eve is wrong.

(a) Explain Eve's mistake. [2 marks]



(b) Find P. [3 marks]

Answer .....

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(c) The particle is moved to a point D on the rod. The force at B keeps the same direction and its magnitude is now $50\ \mathrm{N}$. Find the distance AD. [3 marks]

Answer .....

Total for D2: 8 marks

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No route is given. The hints are at the end of the sheet. Use one at a time.

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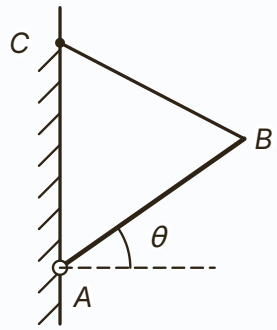
E1 A uniform rod AB has weight W and length $2a$. The end A is freely hinged to a vertical wall. A light inextensible string joins B to the point C on the wall vertically above A, where $AC = 2a$. The rod rests in equilibrium in a vertical plane perpendicular to the wall, with B above the level of A and AB at an angle θ to the horizontal, where $0 < \theta < 90°$. Show that the tension in the string is $W \sin\left(45° - \frac{1}{2}\theta\right)$. The string breaks if the tension is greater than $0.6W$. Find the set of values of θ for which the rod can rest in this way. [9 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Triangle ACB is isosceles with $AC = AB = 2a$. Find the angle CAB in terms of θ, and then the angles at B and C.

Hint 2 Take moments about A. The perpendicular distance from A to the string is $2a\cos\left(\frac{1}{2}\angle CAB\right)$; the weight has perpendicular distance $a\cos\theta$.

Hint 3 Use $\sin 2x = 2\sin x\cos x$ with $\cos\theta = \sin(90^\circ - \theta)$. For the inequality, $\sin$ is increasing on the interval you need.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write the two conditions for a rigid body to be in equilibrium.	○ ○ ○
I can choose a point to take moments about so that an unknown force has no moment.	○ ○ ○
I can find reactions and tensions, and the position of a load, for a rod on supports or strings.	○ ○ ○
I can solve problems on hinged and inclined rods using moments about the hinge or the end.	○ ○ ○

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