Projectile motion formulae

126 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-year-2-projectile-motion-formulae/

Date

A Do Now

Quick questions to start. Some use earlier work; some set a slip that costs marks.

A1 Last lesson [3 marks]

A particle is projected from horizontal ground with speed $19.6\ \text{m s}^{-1}$ at 30° above the horizontal. Find the time of flight and the range.

Answer .....

A2 Common slip [2 marks]

Given that $\sin\alpha = 0.6$ and α is acute, find the value of $\sin 2\alpha$.

Answer .....

A3 Needed today [3 marks]

Solve $3\tan^2\theta - 4\tan\theta + 1 = 0$ for $0^\circ < \theta < 90^\circ$.

Answer .....

A4 Needed today [2 marks]

The position of a particle at time t is given by $x = 3t$ and $y = 4t - 5t^2$. Eliminate t to find y in terms of x.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A particle is projected from a point O on horizontal ground with speed U m s^{-1} at an angle α above the horizontal. Show that the time of flight is $\dfrac{2U\sin\alpha}{g}$. Hence find the time of flight when $U = 21$ and $\alpha = 40°$.



Hint Work with the vertical motion only: $u = U\sin\alpha$, $a = -g$. On level ground the particle lands when $s = 0$.

B1 A particle is projected from level ground with speed U at an angle α above the horizontal. Show that it reaches its greatest height after $\dfrac{U\sin\alpha}{g}$ and that the greatest height is $\dfrac{U^2\sin^2\alpha}{2g}$. Hence find both when $U = 28$ and $\alpha = 60°$. [6 marks]

Answer .....

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EXAMPLE 2

A particle is projected from horizontal ground with speed U at an angle α. Show that its range is $R = \dfrac{U^2 \sin 2\alpha}{g}$. Deduce the greatest possible range for a given U, and find it when $U = 35$.

Hint Range $=$ horizontal component $\times$ time of flight. Then use $2\sin\alpha\cos\alpha = \sin 2\alpha$. The largest value of $\sin 2\alpha$ is 1.

B2 A particle is projected from horizontal ground with speed $24.5\ \text{m s}^{-1}$ and lands $49\ \text{m}$ away. Find the two possible angles of projection. [5 marks]

Answer .....

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EXAMPLE 3

A particle is projected from horizontal ground. Its greatest height is $20\ \mathrm{m}$ and its range is $80\ \mathrm{m}$. Find the angle and the speed of projection.

Hint Divide the formula for the greatest height by the formula for the range: U^2 and g cancel.

B3 A particle is projected from horizontal ground. Its greatest height is $15\ \mathrm{m}$ and its range is $40\ \mathrm{m}$. Find the angle and the speed of projection. [5 marks]

Answer .....

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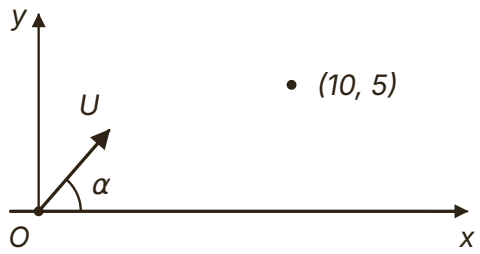
EXAMPLE 4

A particle is projected from a point O with speed U at an angle α above the horizontal. (a) Show that the equation of its path is $y = x\tan\alpha - \dfrac{gx^2}{2U^2}(1+\tan^2\alpha)$, where x and y are measured horizontally and vertically upwards from O. (b) Given $U = 14\ \text{m s}^{-1}$ and that the particle passes through the point $(10, 5)$, find the two possible values of α.



Hint From $x = U\cos\alpha\, t$, $t = \dfrac{x}{U\cos\alpha}$. Put this into $y = U\sin\alpha\, t - \frac{1}{2}gt^2$, then use $\dfrac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$.

B4 A particle is projected from a point O with speed $21\ \text{m s}^{-1}$ at an angle α above the horizontal and passes through the point $30\ \text{m}$ horizontally from O and $10\ \text{m}$ above it. Find the two possible values of α. Show that 9.8 is used in your working. [6 marks]

Answer .....

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EXAMPLE 5

The path of a particle projected from O is $y = 0.75x - \dfrac{x^2}{40}$, with x and y in metres measured horizontally and vertically upwards from O, and $g = 9.8$. Find the speed of projection, the range on the horizontal through O, and the greatest height.

Hint Compare with $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$. The coefficient of x is $\tan\alpha$.

B5 [6 marks]

The path of a particle projected from O is $y = x\sqrt{3} - \dfrac{x^2}{30}$, in the same units and with $g = 9.8$. Find the speed and angle of projection, the range on the horizontal through O, and the greatest height.

Answer .....

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C1 A ball is thrown from a point $2\ \text{m}$ above horizontal ground with speed $14\ \text{m s}^{-1}$ at 45° above the horizontal. Ella says: "The range is $\dfrac{14^2 \sin 90^\circ}{9.8} = 20\ \text{m}$." Explain why Ella's method is not valid here, and find the horizontal distance the ball travels before it hits the ground. [5 marks]

Answer .....

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C2 A particle is projected from O with speed $28\ \text{m s}^{-1}$ and passes through the point $(40, 10)$. Sam writes: "$10 = 40\tan\alpha - \dfrac{9.8 \times 1600}{2 \times 784}\tan^2\alpha$, so $\tan^2\alpha - 4\tan\alpha + 1 = 0$." Explain Sam's mistake and find the correct possible angles of projection. [5 marks]

Answer .....

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C3 [5 marks]

The path of a ball is $y = 0.5x - \dfrac{x^2}{80}$, with x and y in metres measured horizontally and vertically upwards from the point of projection, and $g = 9.8$. Lily says: "The greatest height is where $0.5 - \dfrac{x}{40} = 0$, so the greatest height is $20\ \text{m}$." Explain Lily's mistake. Find the greatest height and the speed of projection.

Answer .....

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C4 Two balls are projected from the same point on horizontal ground with speed $21\ \mathrm{m\ s^{-1}}$, one at 30° and the other at 60° above the horizontal. Josh says the ball at 60° lands further away because it is in the air for longer. Explain why Josh is wrong and find the range of each ball. [4 marks]

Answer .....

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C5 Sam says: "To throw a ball as far as possible, always throw it at $45°$." (i) Show that, in the model of a particle projected from level ground with no air resistance, the range is greatest at $45°$. (ii) Give two reasons why a real throw might not go furthest at $45°$. [4 marks]

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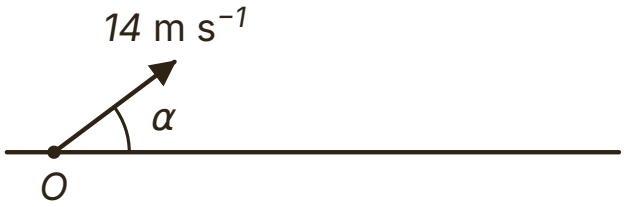
D1 A ball is kicked from a point O on horizontal ground with speed $14\ \text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$. The ball is modelled as a particle moving freely under gravity. Take x and y in metres, horizontally and vertically upwards from O.

(a) Show that the equation of the path of the ball is $y = 0.75x - \dfrac{x^2}{25.6}$. [4 marks]



Answer .....

(b) Find the range of the ball. [2 marks]

Answer .....

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(c) Find the greatest height of the ball. [3 marks]

Answer .....

(d) A fence $2\ \text{m}$ high stands $15\ \text{m}$ from O, in the plane of the motion. Determine whether the ball passes over the fence. [2 marks]

Answer .....

Total for D1: 11 marks

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D2

A particle is projected from horizontal ground with speed $28\ \text{m s}^{-1}$ and lands $60\ \text{m}$ away. Nina writes: "$\sin 2\alpha = \dfrac{60 \times 9.8}{28^2} = 0.75$, so $2\alpha = 48.6^\circ$ and the angle of projection is 24.3°."

Nina has not found all the possible angles.

(a) Explain what Nina has missed and find every possible angle of projection. [4 marks]

Answer .....

(b) Find the time of flight for each angle. [3 marks]

Answer .....

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(c) Find the greatest height for each angle. [3 marks]

Answer .....

Total for D2: 10 marks

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D3 A ball is kicked from a point O on horizontal ground with speed $U\ \mathrm{m\ s^{-1}}$. There are two angles of projection for which the ball lands at a point $40\ \mathrm{m}$ from O. For one of these angles the time of flight is twice the time of flight for the other. The ball is modelled as a particle moving freely under gravity. Find the value of U. [7 marks]

Answer .....

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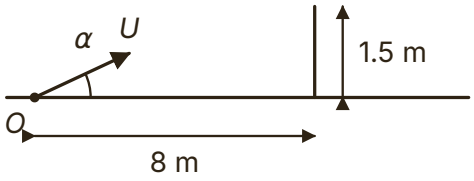
D4 A ball is hit from a point O on horizontal ground with speed $U\ \mathrm{m\ s^{-1}}$ at an angle α above the horizontal. It just passes over the top of a net, which is $1.5\ \mathrm{m}$ high and $8\ \mathrm{m}$ from O, measured horizontally. The ball is modelled as a particle moving freely under gravity.



(a) Show that $1.5 = 8\tan\alpha - \dfrac{313.6}{U^2\cos^2\alpha}$. [4 marks]

Answer .....

(b) Given that $\alpha = 25°$, find U. [3 marks]

Answer .....

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(c) Find the speed of the ball as it passes over the net, and state whether it is rising or falling. [4 marks]

Answer .....

Total for D4: 11 marks

D5 A ball is kicked from a point O on horizontal ground with speed $U\ \mathrm{m\ s^{-1}}$ at an angle α above the horizontal. It must land at a point A on the ground, $30\ \mathrm{m}$ from O. The ball is modelled as a particle moving freely under gravity.

(a) Find the least possible value of U. [3 marks]

Answer .....

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(b) The ball is kicked with this least speed and lands at A. Find the time of flight and the greatest height of the ball. [4 marks]

Answer .....

(c) A real kick at this speed and angle lands short of A. Give a reason for this, and state one way in which the model could be refined. [2 marks]

Answer .....

Total for D5: 9 marks

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D6 A particle is projected from horizontal ground with speed U at an angle α above the horizontal. Its range is R and its greatest height is H.

(a) Given that $R = kH$, show that $\tan\alpha = \dfrac{4}{k}$. [4 marks]

Answer .....

(b) Given that $U = 20\ \text{m s}^{-1}$ and $R = 3H$, find the range. [3 marks]

Answer .....

Total for D6: 7 marks

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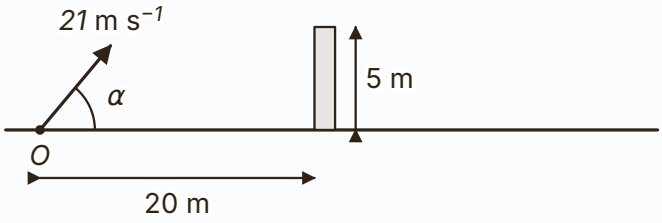
No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A ball is thrown from a point O on horizontal ground with speed $21\ \text{m s}^{-1}$ at an angle α above the horizontal. It must pass over a wall $5\ \text{m}$ high whose foot is $20\ \text{m}$ from O, and it must land no more than $40\ \text{m}$ from O. Find the set of possible values of α. [10 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Use the equation of the path with $x = 20$: the ball is above the wall when $y > 5$. Write it as a quadratic inequality in $T = \tan\alpha$.

Hint 2 $\dfrac{9.8 \times 400}{2 \times 441} = \dfrac{40}{9}$. The wall condition becomes $8T^2 - 36T + 17 < 0$.

Hint 3 For the landing condition use $R = \dfrac{441 \sin 2\alpha}{9.8} \leq 40$: $\sin 2\alpha$ has two ranges of α. Then combine the two conditions.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can derive the time of flight, the greatest height and the range of a projectile.	○ ○ ○
I can derive the equation of the path and use it to find angles of projection.	○ ○ ○
I can find the speed and angle of projection from the range, the greatest height or the path.	○ ○ ○
I can say when the formulae can be used and when they cannot.	○ ○ ○

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