Projection at any angle

124 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-year-2-projection-at-any-angle/

Date

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

A Do Now

Quick questions to start. Some use earlier work; some set a slip that costs marks.

A1 Last lesson [3 marks]

A particle is projected horizontally with speed $7\ \text{m s}^{-1}$ from a point $4.9\ \text{m}$ above horizontal ground. Find the time it takes to reach the ground and the horizontal distance it travels.

Answer .....

A2 Needed today [2 marks]

A velocity of $15\ \text{m s}^{-1}$ is at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Find its horizontal and vertical components.

Answer .....

A3 Needed today [3 marks]

A ball is thrown vertically upwards with speed $19.6\ \text{m s}^{-1}$. Find its greatest height above the point of projection and the time it takes to return to that point.

Answer .....

A4 Common slip [3 marks]

A stone is thrown vertically upwards with speed $14.7\ \text{m s}^{-1}$ from a point $19.6\ \text{m}$ above the ground. Find the time it takes to reach the ground.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

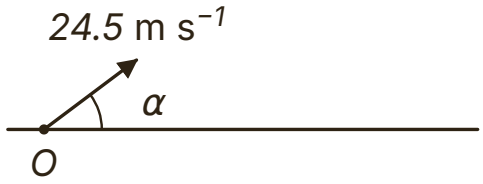
B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A particle is projected from a point O on horizontal ground with speed $24.5\ \text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$. Find (a) the time to reach the greatest height, (b) the greatest height, (c) the time of flight, (d) the range.



Hint Resolve first: $24.5\cos\alpha$ and $24.5\sin\alpha$. At the greatest height the VERTICAL component of the velocity is zero. On level ground the particle lands when the vertical displacement is 0.

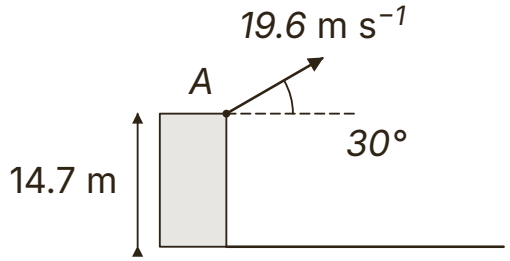
B1 A particle is projected from a point on horizontal ground with speed $35\ \text{m s}^{-1}$ at 30° above the horizontal. Find the time to reach the greatest height, the greatest height, the time of flight and the range. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2

A ball is thrown from a point A, $14.7\ \text{m}$ above horizontal ground, with speed $19.6\ \text{m s}^{-1}$ at 30° above the horizontal. Find the time the ball takes to reach the ground and the horizontal distance it travels.



Hint The ball lands BELOW A, so with up positive its vertical displacement is -14.7. The time is not twice the time to the top.

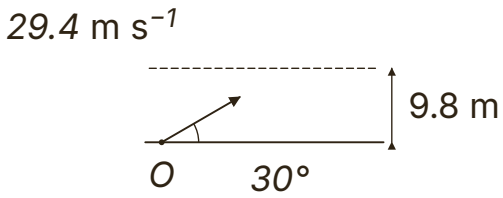
B2 A particle is projected from a point $20\ \text{m}$ above horizontal ground with speed $15\ \text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Find the time it takes to reach the ground and the horizontal distance it travels. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 3

A particle is projected from a point O on horizontal ground with speed $29.4\ \text{m s}^{-1}$ at 30° above the horizontal. Find the times at which it is $9.8\ \text{m}$ above the ground, and the horizontal distance between its two positions at that height.



Hint Set the vertical displacement equal to 9.8. The quadratic has two positive roots: one on the way up, one on the way down.

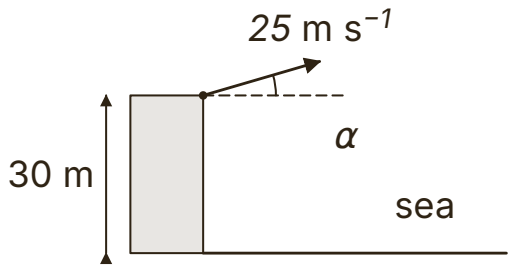
B3 A particle is projected from a point on horizontal ground with speed $20\ \text{m s}^{-1}$ at 60° above the horizontal. Find the length of time for which it is more than $10\ \text{m}$ above the ground, and the horizontal distance it travels in that time. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4

A stone is thrown from the top of a cliff, $30\ \text{m}$ above the sea, with speed $25\ \text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{7}{24}$. Find (a) the speed and direction of the stone as it hits the sea, (b) the direction of motion $1\ \text{s}$ after it is thrown, saying whether it is rising or falling.



Hint $\cos\alpha = \frac{24}{25}$, $\sin\alpha = \frac{7}{25}$. The horizontal component stays 24. For the vertical component use $v_y = 7 - 9.8t$: a negative value means the stone is moving down.

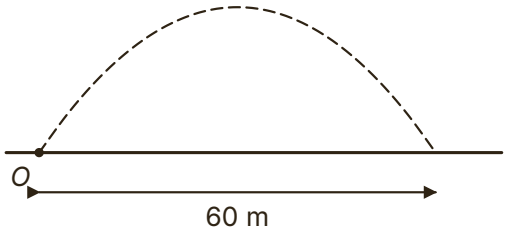
B4 A ball is thrown from a point $18\ \text{m}$ above horizontal ground with speed $13\ \text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$. Find the speed and direction of the ball as it hits the ground. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 5

A golf ball is hit from a point O on horizontal ground. It lands on the ground $4\ \text{s}$ later at a point $60\ \text{m}$ from O. Find the speed and the angle of projection.



Hint Work the two directions separately. The horizontal motion gives $U\cos\alpha$. On level ground the vertical displacement is 0 after $4\ \text{s}$, which gives $U\sin\alpha$.

B5 A particle is projected from a point on horizontal ground. Its greatest height is $19.6\ \text{m}$ and it lands $48\ \text{m}$ from the point of projection. Find the speed and the angle of projection. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 A particle is projected with speed $9.8\ \text{m s}^{-1}$ at 30° above the horizontal from a point $9.8\ \text{m}$ above horizontal ground. Max says: "The time of flight is $\dfrac{2 \times 9.8 \sin 30^\circ}{9.8} = 1\ \text{s}$." Explain Max's mistake. Find the time the particle takes to reach the ground and the horizontal distance it travels. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C2 A ball is thrown with speed $20\ \text{m s}^{-1}$ at $35°$ above the horizontal from a point $3\ \text{m}$ above horizontal ground. Zoe writes: "Greatest height $= \dfrac{(20\cos 35°)^2}{2 \times 9.8} = 13.7\ \text{m}$." Explain Zoe's two mistakes and find the greatest height of the ball above the ground. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C3 A particle is projected from a point on horizontal ground with speed $30\ \text{m s}^{-1}$ at 50° above the horizontal. Raj says: "When it lands, its speed is $30 \sin 50^\circ = 23.0\ \text{m s}^{-1}$." Explain Raj's mistake and give the speed and direction of the particle as it lands. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C4 A particle is projected from horizontal ground with speed $26\ \mathrm{m\ s^{-1}}$ at an angle α above the horizontal, where $\tan\alpha = \frac{12}{5}$. Mia says its speed at the greatest height is zero. Explain why Mia is wrong. Find the speed of the particle and its height at that instant. [4 marks]

Answer .....

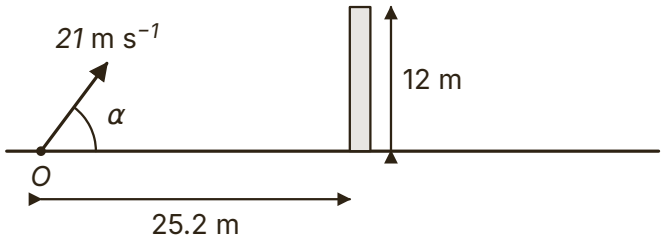
© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C5 A stone is thrown from a point on horizontal ground and is modelled as a particle moving freely under gravity. The model gives a greatest height of H metres. In a refined model air resistance is included, and the greatest height is K metres. (i) State, with a reason, how K compares with H. (ii) State two other limitations of the original model. [4 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D1 A ball is kicked from a point O on horizontal ground with speed $21\ \text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. A vertical wall $12\ \text{m}$ high stands $25.2\ \text{m}$ from O, measured horizontally, in the vertical plane of the motion. The ball is modelled as a particle moving freely under gravity.

(a) Show that the ball passes over the wall, and find by how much it clears the wall. [4 marks]



Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the distance beyond the wall at which the ball lands. [3 marks]

Answer .....

(c) Find the speed and direction of the ball as it passes over the wall. [3 marks]

Answer .....

Total for D1: 10 marks

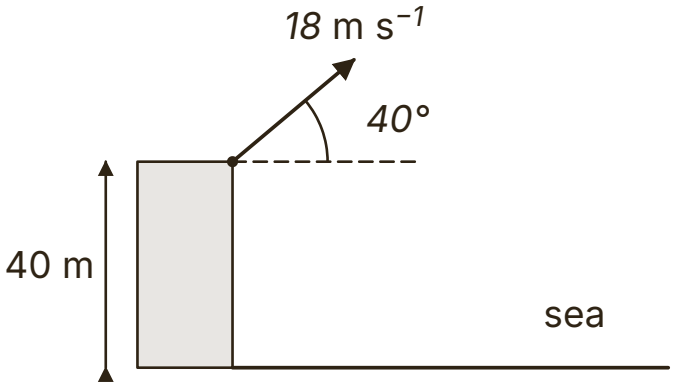
© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D2

A stone is thrown from the top of a cliff $40\ \text{m}$ above the sea with speed $18\ \text{m s}^{-1}$ at $40°$ above the horizontal. Kai writes: "Greatest height $= \dfrac{(18\sin 40°)^2}{2 \times 9.8} = 6.83\ \text{m}$, so the stone is $6.83\ \text{m}$ above the sea at its highest point. Time of flight $= \dfrac{2 \times 18\sin 40°}{9.8} = 2.36\ \text{s}$."

Kai has made two mistakes.

(a) Explain Kai's first mistake and find the greatest height of the stone above the sea. [3 marks]



Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Explain Kai's second mistake. Find the time the stone takes to reach the sea and the horizontal distance it travels. [6 marks]

Answer .....

Total for D2: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

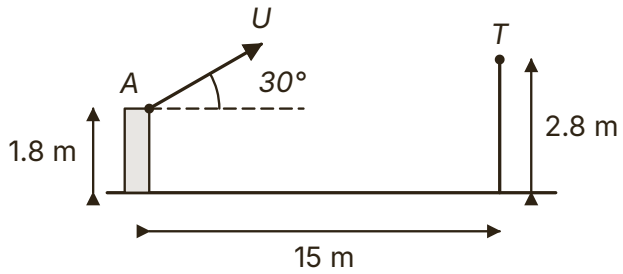
D3 A particle P is projected from a point O on horizontal ground. It is $12.25\ \text{m}$ above the ground at $t = 1\ \text{s}$ and again at $t = 2.5\ \text{s}$ after projection. The horizontal distance between these two positions of P is $24\ \text{m}$. Find the speed and the angle of projection of P. [8 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D4 A ball is thrown from a point A, $1.8\ \text{m}$ above horizontal ground, with speed $U\ \text{m s}^{-1}$ at 30° above the horizontal. It hits a small target T which is $2.8\ \text{m}$ above the ground and $15\ \text{m}$ from A, measured horizontally. The ball is modelled as a particle moving freely under gravity.

(a) Find the value of U. [5 marks]



Answer .....

(b) Find the time the ball takes to reach T, and state whether the ball is rising or falling when it hits T. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) State, with a reason, whether a ball thrown in practice would need a speed greater than or less than your answer to (a) to hit T. [2 marks]

Total for D4: 10 marks

D5 A particle P is projected from a point on horizontal ground with speed $30\ \mathrm{m\ s^{-1}}$ at 60° above the horizontal.

(a) Find the two times at which P is moving at 45° to the horizontal. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the distance between the two positions of P at these times. [4 marks]

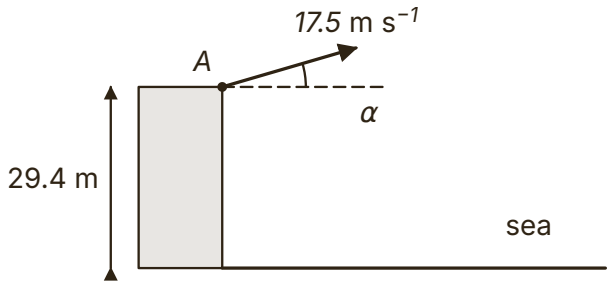
Answer .....

Total for D5: 8 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D6 A stone is thrown from a point A on the top of a cliff, $29.4\ \text{m}$ above the sea, with speed $17.5\ \text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{7}{24}$.

(a) Find the length of time for which the stone is above the level of A. [3 marks]



Answer .....

(b) Find the speed of the stone at the instant when it is $10\ \text{m}$ below the level of A. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) The stone hits the sea at the point B. Find the angle of depression of B from A. [3 marks]

Answer .....

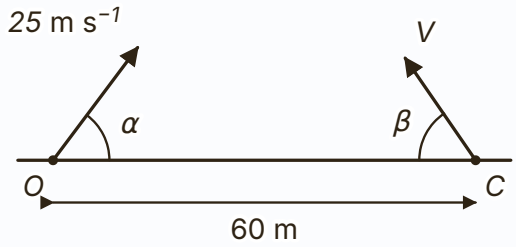
Total for D6: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [9 marks]

Two points O and C are on horizontal ground, $60\ \text{m}$ apart. A particle A is projected from O towards C with speed $25\ \text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. At the same instant a particle B is projected from C towards O with speed $V\ \text{m s}^{-1}$ at an angle β above the horizontal. The particles move in the same vertical plane and collide at the instant when A is at its greatest height. Find V and β, and the speed of B just before the collision.



Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Start with A alone: its components are 15 and 20. Find when and where it reaches its greatest height.

Hint 2 B must be at the same height at the same time as A, from the same starting level. What does that tell you about the vertical component of B's velocity?

Hint 3 The two horizontal distances travelled add up to $60\ \text{m}$. At the collision both vertical components are zero.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the greatest height, the time of flight and the range of a projectile.	○ ○ ○
I can solve problems where a particle is projected at an angle from a point above the ground.	○ ○ ○
I can find the speed and direction of a projectile at any time, and say whether it is rising or falling.	○ ○ ○
I can find the speed and angle of projection from information about the path.	○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.