Static rigid bodies

101 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-year-2-static-rigid-bodies/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks.

A1 Needed today [2 marks]

A light rod PQ of length $4\ \text{m}$ is pivoted at P. A force of $10\ \text{N}$ acts at Q at an angle of 30° to the rod. Find the moment of the force about P.

Answer .....

A2 Last lesson [2 marks]

A small block rests on a rough plane inclined at 28° to the horizontal, with no other force acting, and is on the point of slipping. Find the coefficient of friction.

Answer .....

A3 Needed today [3 marks]

A uniform beam AB of length $6\ \text{m}$ and mass $20\ \text{kg}$ rests horizontally on two supports, one at A and one at C, where $AC = 4\ \text{m}$. Find the reaction at each support.

Answer .....

A4 Common slip [2 marks]

A uniform rod AB of length $5\ \text{m}$ and weight $40\ \text{N}$ has A on horizontal ground and makes an angle of 60° with the ground. Find the moment of the weight about A.

Answer .....

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PART 1 A ladder against a smooth wall

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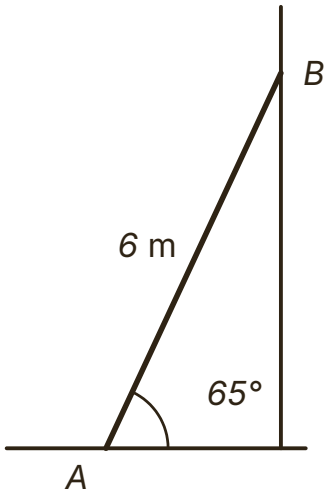
EXAMPLE 1 · I DO

A uniform ladder of length $6\ \mathrm{m}$ and mass $15\ \mathrm{kg}$ rests with its foot A on rough horizontal ground and its top B against a smooth vertical wall. The ladder makes an angle of 65° with the ground and is on the point of slipping. Find the reaction at the wall and the coefficient of friction between the ladder and the ground.



Hint Take moments about A: then R and F at the foot have no moment, and only S and the weight are left.

YOUR TURN · WE DO

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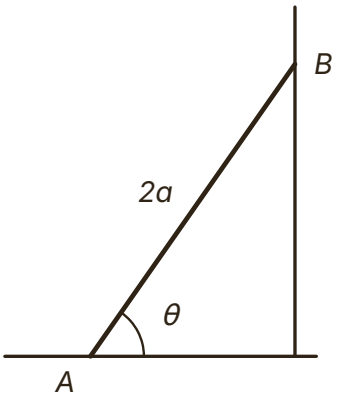
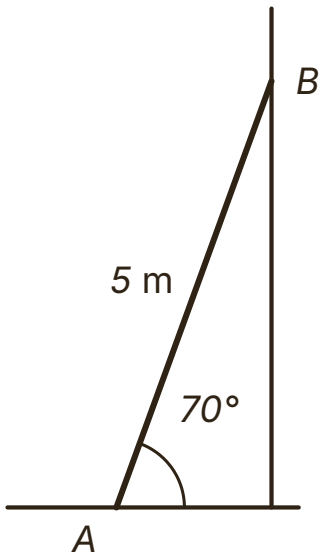
PRACTICE · YOU DO

C1 A uniform ladder of length $2a$ and weight W rests with its foot on rough horizontal ground and its top against a smooth vertical wall, at an angle θ to the ground. The coefficient of friction at the ground is μ. Show that the ladder can rest in equilibrium only if $\tan\theta \geq \dfrac{1}{2\mu}$. [4 marks]



B1 A uniform ladder of length $5\ \text{m}$ and mass $12\ \text{kg}$ rests with its foot on rough horizontal ground and its top against a smooth vertical wall. It makes an angle of 70° with the ground and is on the point of slipping. Find the reaction at the wall and the coefficient of friction. [5 marks]



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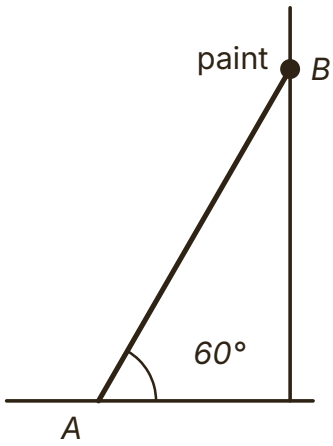
C2 A uniform ladder of length $4\ \mathrm{m}$ and mass $10\ \mathrm{kg}$ rests with its foot on rough horizontal ground and its top against a smooth vertical wall, at 60° to the ground. A can of paint of mass $5\ \mathrm{kg}$ hangs from the top of the ladder. Ravi writes the moment of the ladder's weight about the foot as $10g \times 2\sin 60^\circ$. Explain his error, and find the least value of the coefficient of friction for the ladder to stay at rest. [5 marks]

Answer .....



Answer .....

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PART 2 A ladder with a person on it

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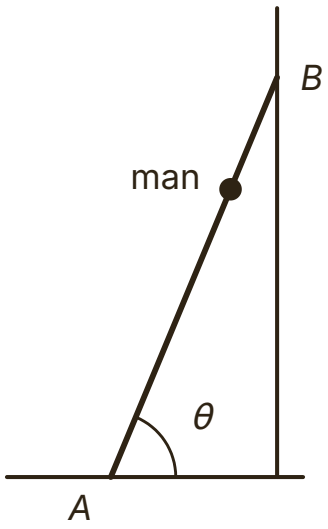
EXAMPLE 2 · I DO

A uniform ladder AB of length $8\ \mathrm{m}$ and mass $20\ \mathrm{kg}$ rests with A on rough horizontal ground and B against a smooth vertical wall, at an angle θ to the ground, where $\tan\theta = \frac{12}{5}$. The coefficient of friction at the ground is 0.35. A man of mass $80\ \mathrm{kg}$ climbs the ladder. Find how far up the ladder he can climb before it slips.



Hint Model the man as a particle a distance x m from A. On the point of slipping, $F = 0.35R$, and R carries both weights.

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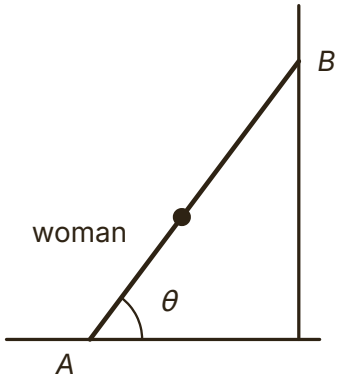
B2 [6 marks]

A uniform ladder of length $6\ \text{m}$ and mass $15\ \text{kg}$ rests with its foot on rough horizontal ground and its top against a smooth vertical wall, at an angle θ to the ground, where $\tan\theta = \frac{4}{3}$. The coefficient of friction at the ground is 0.3. A woman of mass $60\ \text{kg}$ climbs the ladder. Find how far up the ladder she can climb before it slips.



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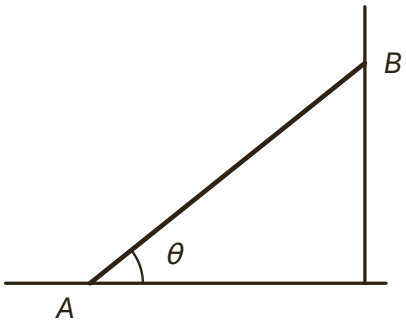
EXAMPLE 3 · I DO

A uniform ladder rests with its foot on rough horizontal ground and its top against a rough vertical wall. The coefficient of friction is 0.5 at the ground and 0.4 at the wall. The ladder is on the point of slipping, making an angle θ with the ground. Find θ.



Hint On the point of slipping, friction is limiting at BOTH ends. The top slides down the wall, so friction at the wall acts upwards.

YOUR TURN · WE DO

Answer .....

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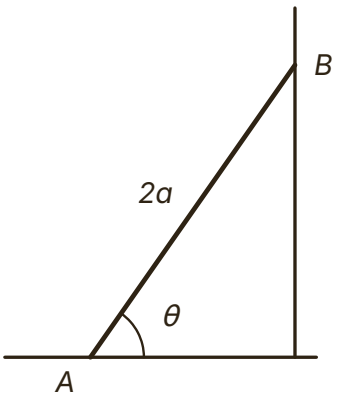
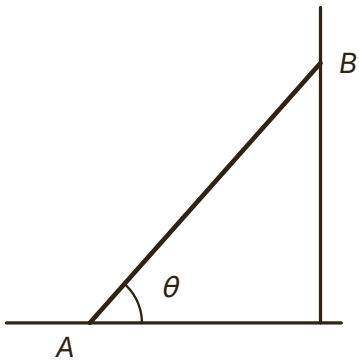
PRACTICE · YOU DO

C3 A uniform ladder of length $2a$ and weight W rests with its foot on rough horizontal ground and its top against a smooth vertical wall, at an angle θ to the ground. The coefficient of friction at the ground is 0.4. A load of weight W is fixed to the top of the ladder. Find the least value of θ for which the ladder can rest in equilibrium. [4 marks]



Answer .....

B3 A uniform ladder rests with its foot on rough horizontal ground and its top against a rough vertical wall. The coefficient of friction is 0.4 at the ground and 0.25 at the wall. The ladder is on the point of slipping, at an angle θ to the ground. Find θ. [6 marks]



Answer .....

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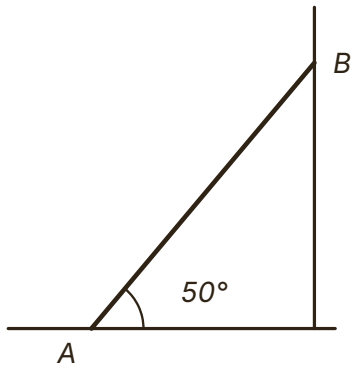
C4 A uniform ladder rests with its foot on rough horizontal ground and its top against a rough vertical wall, where the coefficient of friction is 0.5. The ladder makes an angle of 50° with the ground and is on the point of slipping. Bo draws the friction at the wall acting downwards. Explain why this is wrong, and find the coefficient of friction at the ground. [6 marks]



Answer .....

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PART 3 Rods on pegs and hinged rods

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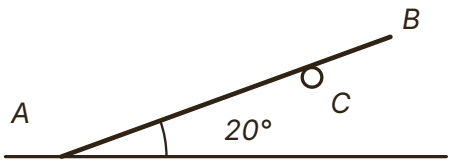
EXAMPLE 4 · I DO

A uniform rod AB of length $4\ \mathrm{m}$ and mass $10\ \mathrm{kg}$ rests with A on rough horizontal ground. The rod rests on a smooth peg at C, where $AC = 3\ \mathrm{m}$, and makes an angle of 20° with the ground. The rod is on the point of slipping. Find the reaction at the peg and the coefficient of friction.



Hint A smooth peg pushes perpendicular to the rod, not vertically. Take moments about A to find it, then resolve.

YOUR TURN · WE DO

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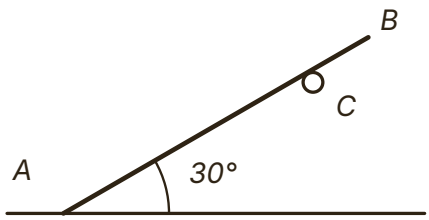
B4 [6 marks]

A uniform rod AB of length $5\ \mathrm{m}$ and mass $8\ \mathrm{kg}$ rests with A on rough horizontal ground and against a smooth peg at C, where $AC = 4\ \mathrm{m}$. The rod makes an angle of 30° with the ground and is on the point of slipping. Find the reaction at the peg and the coefficient of friction.



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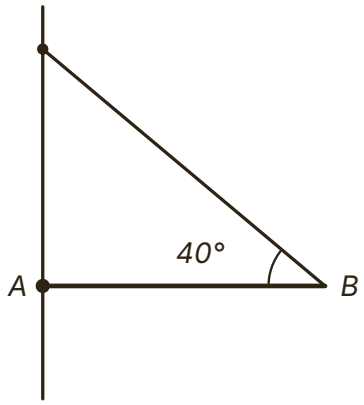
EXAMPLE 5 · I DO

A uniform rod AB of length $2\ \mathrm{m}$ and mass $6\ \mathrm{kg}$ is hinged at A to a vertical wall. It is held horizontal by a light string from B to a point on the wall above A; the string makes an angle of 40° with the rod. Find the tension in the string and the magnitude and direction of the force exerted on the rod by the hinge.



Hint Moments about A remove the hinge force. Then resolve horizontally and vertically to get its two components.

YOUR TURN · WE DO

Answer .....

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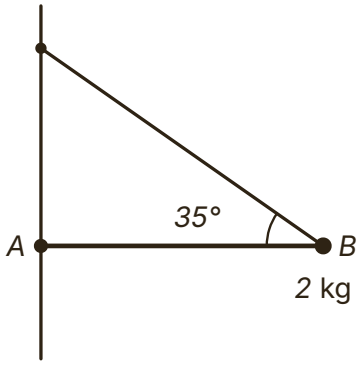
B5 [6 marks]

A uniform rod AB of length $3\ \text{m}$ and mass $4\ \text{kg}$ is hinged at A to a vertical wall. A particle of mass $2\ \text{kg}$ is fixed at B. The rod is held horizontal by a light string from B to a point on the wall above A; the string makes an angle of 35° with the rod. Find the tension, and the magnitude and direction of the force at the hinge.



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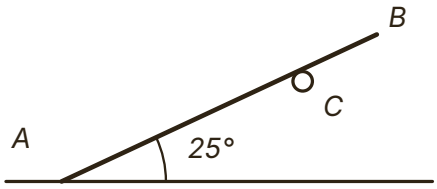
C5 A uniform rod AB of length $6\ \text{m}$ and mass $12\ \text{kg}$ rests with A on rough horizontal ground and against a smooth peg at C, where $AC = 4.5\ \text{m}$. The rod makes an angle of 25° with the ground. Jo draws the peg reaction vertically upwards. Explain her error. Find the reaction at the peg, the friction and the normal reaction at A, and the least possible coefficient of friction. Given $\mu = 0.6$, decide whether the rod is on the point of slipping. [6 marks]

Answer .....



Answer .....

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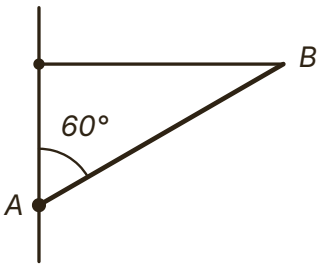
C6 A uniform rod AB of length $3\ \text{m}$ and mass $5\ \text{kg}$ is hinged at A to a vertical wall. The rod points upwards, making an angle of 60° with the wall, and is held in place by a light horizontal string from B to the wall. Find the tension in the string and the magnitude and direction of the force at the hinge. [7 marks]



Answer .....

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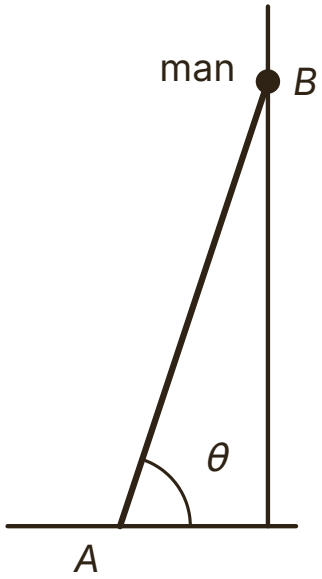
D1

A uniform ladder of length $6\ \text{m}$ and mass $20\ \text{kg}$ rests with its foot on rough horizontal ground and its top against a smooth vertical wall, at an angle θ to the ground, where $\tan\theta = 3$. The coefficient of friction at the ground is 0.25. A man of mass $70\ \text{kg}$ stands at the top. To stop the ladder slipping, a horizontal force $P\ \text{N}$ towards the wall is applied at the foot. Ola says the least value is given by $P = S + \mu R$, where S is the reaction at the wall.

Ola is wrong.

(a) Show that without P the ladder would slip. [4 marks]



(b) Explain Ola's error. [2 marks]

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(c) Find the least value of P. [3 marks]

Answer .....

(d) State, with a reason, the direction of the friction at the foot if P is made very large. [2 marks]

Total for D1: 11 marks

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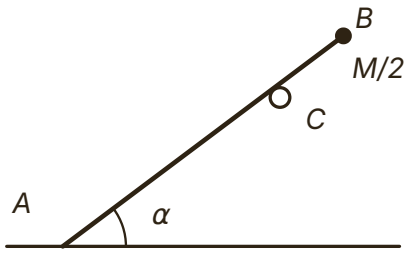
D2 A uniform rod AB of length $4a$ and mass M rests with A on rough horizontal ground and against a smooth peg at C, where $AC = 3a$. The rod makes an angle α with the ground, where $\tan\alpha = \frac{3}{4}$. A particle of mass $\frac{1}{2}M$ is attached at B, and the rod is on the point of slipping.



(a) Show that the reaction at the peg is $\frac{16}{15}Mg$. [4 marks]

(b) Find the coefficient of friction between the rod and the ground. Give your answer to 3 significant figures. [5 marks]

Answer .....

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(c) State how you used the fact that the peg is smooth. [2 marks]

Total for D2: 11 marks

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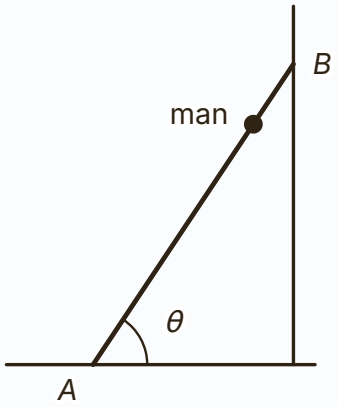
E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A uniform ladder of length $2a$ and weight W rests with its foot on rough horizontal ground and its top against a rough vertical wall. The coefficient of friction is $\frac{1}{2}$ at the ground and $\frac{1}{3}$ at the wall. The ladder makes an angle θ with the ground, where $\tan\theta = 1.5$. A man of weight $4W$ climbs the ladder. Find how far up the ladder he can climb before it slips. [9 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 When it is about to slip, friction is limiting at both ends: $F = \frac{1}{2}R$ towards the wall, and $\frac{1}{3}S$ up the wall.

Hint 2 Resolve both ways to find R and S in terms of W: you should get $R = \frac{30}{7}W$.

Hint 3 Take moments about the foot with the man at distance x, then divide through by $\cos\theta$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can take moments about a well-chosen point and resolve to find the reactions on a ladder or rod.	○ ○ ○
I can find the coefficient of friction, or how far a person can climb, when a ladder is about to slip.	○ ○ ○
I can find the reaction at a peg, perpendicular to the rod, and the friction at the ground.	○ ○ ○
I can find the tension in a string holding a hinged rod and the size and direction of the force at the hinge.	○ ○ ○

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