Variable acceleration in one dimension

141 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-year-2-variable-acceleration/

Date

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A Do Now

Quick questions to start. Some check what you need today; some set a slip that the examiners' reports say costs marks.

A1 Needed today [2 marks]

Find $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 3\mathrm{e}^{-2t} + 4\sin 3t$.

Answer .....

A2 Needed today [3 marks]

Find $\displaystyle\int (6\cos 2t + 4\mathrm{e}^{-2t})\,\mathrm{d}t$.

Answer .....

A3 Needed today [3 marks]

A particle starts at O and moves in a straight line with velocity $v = 3t^2 - 12t + 9\ \mathrm{m\ s^{-1}}$ at time t seconds. Find the total distance it travels in the first 3 seconds.

Answer .....

A4 Common slip [2 marks]

A particle has velocity $v = 5\sin(0.5t)\ \mathrm{m\ s^{-1}}$. Ravi finds the velocity when $t = 2$ and gets $0.0873\ \mathrm{m\ s^{-1}}$. Explain his mistake and find the correct velocity.

Answer .....

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A5 Common slip [2 marks]

A particle has acceleration $a = 6\cos 2t \text{ m s}^{-2}$. Kim writes $v = 12\sin 2t + c$. Correct her working.

Answer .....

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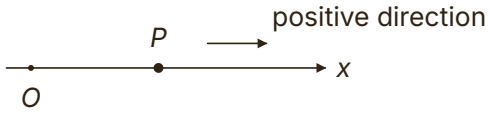
B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A particle of mass $0.5\ \text{kg}$ moves along the positive x-axis. At time t seconds its displacement from O is s metres, where $s = 8\sqrt{t} + 2\text{e}^{-t}$, $t > 0$. Find the velocity when $t = 4$. The particle moves under a single force of variable magnitude F N acting along the x-axis. Find the magnitude and direction of this force when $t = 1$.



Hint Write $8\sqrt{t}$ as $8t^{\frac{1}{2}}$ before you differentiate. The sign of a tells you the direction of the force.

B1 A particle of mass $3\ \text{kg}$ moves along the positive x-axis. At time t seconds its displacement from O is s metres, where $s = 6\sqrt{t} + 4\text{e}^{-0.5t}$, $t > 0$. Find the velocity when $t = 9$, and the magnitude and direction of the resultant force on the particle when $t = 1$. [6 marks]

Answer .....

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EXAMPLE 2

A particle P starts from rest at O and moves in a straight line. At time t seconds its acceleration is $a = 8\sin 2t \text{ m s}^{-2}$. Find v in terms of t, and show that P never moves in the negative direction. Find the greatest speed of P and the first time it reaches it. Find the exact displacement of P from O when $t = \pi$.

Hint Integrate $\sin 2t$ to $-\frac{1}{2}\cos 2t$, then use $v = 0$ when $t = 0$. The identity $1 - \cos 2t = 2\sin^2 t$ shows the sign of v.

B2 A particle P starts from rest at O and moves in a straight line. At time t seconds its acceleration is $a = 6\sin 3t \text{ m s}^{-2}$. Find v in terms of t. Find the greatest speed of P and the first time at which it reaches it. Find the displacement of P from O when $t = \dfrac{\pi}{2}$. [7 marks]

Answer .....

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EXAMPLE 3

A particle P starts at O and moves along a straight line with velocity $v = 1 + 2\sin t \text{ m s}^{-1}$, $0 \leq t \leq 2\pi$. Find the times when P is at rest, state how many times it changes direction, and find the exact total distance it travels and its displacement from O at $t = 2\pi$.

Hint Solve $\sin t = -\frac{1}{2}$ in the interval: both solutions lie where $\sin$ is negative. Find the position at $t = 0$, at each rest and at $t = 2\pi$, then add the sizes of the moves.

B3 A particle P starts at O and moves along a straight line with velocity $v = 1 - 2\cos t \text{ m s}^{-1}$, $0 \leq t \leq \pi$. Find the time when P is at rest. Find, in exact form, the total distance travelled by P in the interval, and its displacement from O at $t = \pi$. [6 marks]

Answer .....

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EXAMPLE 4

A particle P moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = 6t\mathrm{e}^{-\frac{t}{3}}$, $t \geq 0$. Find the greatest displacement of P from O, and the acceleration of P at that instant. Describe what happens to P as t becomes large.

Hint Use the product rule. The greatest displacement is where $v = 0$, not where $a = 0$.

B4 A particle P moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = 8t\mathrm{e}^{-\frac{t}{2}}$, $t \geq 0$. Find the greatest displacement of P from O and the acceleration of P at that instant. State what happens to P as t becomes large. [8 marks]

Answer .....

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EXAMPLE 5

A particle P starts from rest at O and moves in a straight line. At time t seconds its acceleration is $a = \dfrac{6}{(t+1)^2}\ \text{m s}^{-2}$. Find v in terms of t and show that the speed of P is always less than $6\ \text{m s}^{-1}$. Find the distance travelled in the first 3 seconds.

Hint Write $a = 6(t+1)^{-2}$ and integrate as a power. $\displaystyle\int \frac{1}{t+1}\,\mathrm{d}t = \ln(t+1)$.

B5 [7 marks]

A particle P starts from rest at O and moves in a straight line. At time t seconds its acceleration is $a = \dfrac{4}{(t+2)^2}\ \text{m s}^{-2}$. Find v in terms of t and explain why the speed of P never reaches $2\ \text{m s}^{-1}$. Find, in exact form, the distance P travels while its speed increases from 0 to $1.5\ \text{m s}^{-1}$.

Answer .....

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C1 A particle starts from rest and moves in a straight line with acceleration $a = (2 + 6\sin t)\ \text{m s}^{-2}$ at time t seconds. Ben writes: "When $t = 0$, $a = 2$. So after 3 seconds, $v = u + at = 0 + 2 \times 3 = 6\ \text{m s}^{-1}$." Explain why Ben's method is not valid, and find the correct velocity when $t = 3$. [4 marks]

Answer .....

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C2 A particle moves in a straight line with acceleration $a = 12\cos 2t \text{ m s}^{-2}$. When $t = \frac{\pi}{4}$, its velocity is 5 m s^{-1}. Kim writes "$v = 24\sin 2t + c$" and later says "the greatest speed is 5 m s^{-1}". Correct Kim's integration, and find the greatest speed of the particle. [5 marks]

Answer .....

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C3 A particle starts at O and moves in a straight line with velocity $v = 3\cos\left(\frac{\pi t}{2}\right)$ m s^{-1}. Sam writes: "Distance in the first 2 seconds $= \int_0^2 3\cos\left(\frac{\pi t}{2}\right)\,\mathrm{d}t = 0$." Explain why Sam is wrong and find the exact distance travelled in the first 2 seconds. [4 marks]

Answer .....

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C4 A sledge slides in a straight line across flat snow. Its velocity is modelled as $v = 8 - 2\sqrt{t}\ \text{m s}^{-1}$, $0 \le t \le 16$. Find the distance the sledge travels in the 16 seconds. Find its acceleration when $t = 4$. Explain what the model predicts about the acceleration at $t = 0$, and why this is a criticism of the model. [5 marks]

Answer .....

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C5 A cyclist moves along a straight road. Her velocity is modelled as $v = 10 - 6\mathrm{e}^{-\frac{t}{2}}\ \mathrm{m\,s^{-1}}$, $t \geq 0$. Jo says: "If she rides for long enough, she will reach $10\ \mathrm{m\,s^{-1}}$." Show that Jo is wrong, and show that the cyclist is always speeding up. Find the distance she travels in the first 4 seconds. [5 marks]

Answer .....

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D1 A particle P moves in a straight line. At time t seconds, $t \geq 0$, its velocity is v m s^{-1} and its acceleration is a m s^{-2}, where $v = 3 + 5\sin kt$ and k is a positive constant. The initial acceleration of P is 10 m s^{-2}.

(a) Find the value of k. [3 marks]

Answer .....

(b) Show that $a^2 = 100 - 4(v-3)^2$. [3 marks]

Answer .....

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(c) Hence find the possible speeds of P at an instant when its acceleration is $6\ \mathrm{m\ s^{-2}}$. [3 marks]

Answer .....

(d) Find the time at which P is first at rest. [3 marks]

Answer .....

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(e) Find the distance travelled by P from $t = 0$ until it is first at rest. [4 marks]

Answer .....

Total for D1: 16 marks

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D2 A particle P starts at O and moves along the positive x-axis. At time t seconds its velocity is $v\ \mathrm{m\ s^{-1}}$, where

$$v = \begin{cases} 4\sqrt{t}, & 0 \le t \le 4 \\ 8\cos\left(\dfrac{\pi(t-4)}{8}\right), & t > 4 \end{cases}$$

(a) Show that the two expressions for v agree when $t = 4$, and find the acceleration of P just before and just after $t = 4$. [4 marks]

Answer .....

(b) Find the distance of P from O when $t = 4$. [3 marks]

Answer .....

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(c) Find the value of t at which P is first instantaneously at rest. [2 marks]

Answer .....

(d) Find the total distance travelled by P in the first 12 seconds, and its displacement from O when $t = 12$. [6 marks]

Answer .....

Total for D2: 15 marks

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D3

> Lily writes: "$a = \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\dfrac{5}{(t+1)^2}$, which is never zero. So x has no greatest value: P moves further from O for ever."

A particle P of mass $2\ \mathrm{kg}$ moves along the x-axis. At time t seconds, $t \geq 0$, its displacement from O is x metres, where $x = 5\ln(t+1) - t$.

(a) Explain the mistake in Lily's working. [2 marks]

(b) Find the greatest value of x. [3 marks]

Answer .....

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(c) Find the magnitude and direction of the resultant force on P at the instant when x is greatest. [3 marks]

Answer .....

(d) Show that P passes back through O at a time between $t = 13$ and $t = 14$. [3 marks]

Answer .....

Total for D3: 11 marks

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D4 A particle P moves along a straight line. At time t seconds, $t > 0$, its acceleration is $\left(1 - \frac{2}{\sqrt{t}}\right)\ \mathrm{m\,s^{-2}}$. When $t = 4$, the velocity of P is $-1\ \mathrm{m\,s^{-1}}$. P is instantaneously at rest at two different times. Find the distance between the two points where P is at rest. [8 marks]

Answer .....

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D5 A car of mass $1200\ \text{kg}$ moves along a straight horizontal road. At time $t = 0$ the driver applies the brakes. At time t seconds after this, the velocity of the car is modelled as $v = (30\text{e}^{-\frac{t}{5}} - 6)\ \text{m s}^{-1}$, $0 \leq t \leq T$, where T is the time at which the car comes to rest.

(a) Find the exact value of T. [3 marks]

Answer .....

(b) Find the magnitude of the resultant force on the car when $t = 0$, and explain why this force decreases as the car slows down. [4 marks]

Answer .....

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(c) Find the distance travelled by the car while it is braking. [4 marks]

Answer .....

(d) Explain why the model is only used for $0 \leq t \leq T$, and state one limitation of the model. [2 marks]

Total for D5: 13 marks

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No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 A particle P moves in a straight line. At time t seconds, $t \geq 0$, its velocity is $v = 4 + k\sin\left(\frac{\pi t}{6}\right)$ $\mathrm{m\,s^{-1}}$, where k is a positive constant. Find the set of values of k for which P never changes its direction of motion. In the case $k = 8$, find the greatest speed of P, and the exact total distance P travels in the first 12 seconds. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $\sin$ lies between -1 and 1. The particle changes direction only if v becomes negative, so find the least value of v.

Hint 2 For $k = 8$, solve $v = 0$ for $0 \leq t \leq 12$: there are two solutions, so there are three stages of motion.

Hint 3 Integrate v, find the position at $t = 0$, at each rest and at $t = 12$, and add the sizes of the three moves.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can differentiate and integrate trig, exponential, root and reciprocal functions of time to move between s, v and a.	○ ○ ○
I can find constants of integration from given conditions and the greatest speed of a particle.	○ ○ ○
I can find when a particle is at rest and the total distance it travels when it changes direction.	○ ○ ○
I can use F = ma with a variable acceleration and criticise a model of motion.	○ ○ ○

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