Vector methods with projectiles

140 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-year-2-vector-methods-projectiles/

Date

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A Do Now

Quick questions to start. Some check what you need today; some set a slip that costs marks.

A1 Needed today [3 marks]

A particle starts at the point with position vector $(\mathbf{i} + 2\mathbf{j})$ m with velocity $(4\mathbf{i} - 3\mathbf{j})$ m s^{-1} and moves with constant acceleration $(2\mathbf{i} + 5\mathbf{j})$ m s^{-2}. Find its position vector and its speed when $t = 2$.

Answer .....

A2 Needed today [3 marks]

A ball is projected with speed 20 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$. Write its velocity of projection in the form $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, and find the time it takes to reach its greatest height.

Answer .....

A3 Common slip [3 marks]

A particle is projected from O with velocity $(5\mathbf{i} + 14\mathbf{j})$ m s^{-1}. Mo writes $\mathbf{v} = (5\mathbf{i} + 14\mathbf{j}) + 2(9.8\mathbf{j})$ for its velocity when $t = 2$. Find the correct velocity and speed when $t = 2$.

Answer .....

A4 Common slip [2 marks]

A ball is projected from level ground with velocity $(6\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Ria says the time to the greatest height is $\frac{10}{9.8} = 1.02$ s, because the speed is 10 m s^{-1}. Find the correct time.

Answer .....

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A5 Needed today [2 marks]

Solve $4.9t^2 - 9.8t - 14.7 = 0$ for $t > 0$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

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EXAMPLE 1

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle is projected from a point O on horizontal ground with velocity $(6\mathbf{i} + 14.7\mathbf{j})\ \mathrm{m\ s^{-1}}$.
(a) Find the velocity of the particle and its speed when $t = 2$.
(b) Find the greatest height of the particle above the ground.
(c) Find the distance from O of the point where the particle lands.

Hint Write $\mathbf{r}$ and $\mathbf{v}$ as vectors with $\mathbf{g} = -9.8\mathbf{j}$. The top is where the $\mathbf{j}$-component of the velocity is 0; landing is where the $\mathbf{j}$-component of the position is 0.

B1 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle is projected from a point O on horizontal ground with velocity $(8\mathbf{i} + 19.6\mathbf{j})\ \mathrm{m\ s^{-1}}$.
(a) Find the velocity of the particle and its speed when $t = 3$.
(b) Find the greatest height of the particle above the ground.
(c) Find the distance from O of the point where the particle lands.

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Answer .....

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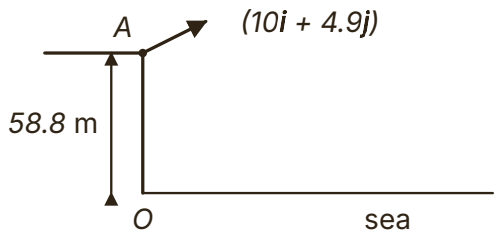
EXAMPLE 2

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A stone is projected from a point A at the top of a vertical cliff with velocity $(10\mathbf{i} + 4.9\mathbf{j})\ \text{m s}^{-1}$. The point O is at sea level vertically below A, and $OA = 58.8\ \text{m}$. The stone hits the sea at the point B.

(a) Find the time taken to reach B and the distance OB.

(b) Find the speed of the stone as it hits the sea and the angle its direction of motion makes with the horizontal.



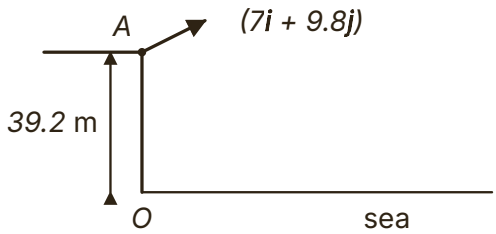
Hint Put the start point into the position vector: $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{g}t^2$. The sea is where the $\mathbf{j}$-component is 0.

B2 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A stone is projected from a point A at the top of a vertical cliff with velocity $(7\mathbf{i} + 9.8\mathbf{j})\ \text{m s}^{-1}$. The point O is at sea level vertically below A, and $OA = 39.2\ \text{m}$. The stone hits the sea at the point B.

(a) Find the time taken to reach B and the distance OB.

(b) Find the speed of the stone as it hits the sea and the angle its direction of motion makes with the horizontal.



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Answer .....

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EXAMPLE 3

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle P is projected from a point O with velocity $(12\mathbf{i} + 16\mathbf{j})\ \text{m s}^{-1}$.

(a) At time T seconds the velocity of P is $(12\mathbf{i} - 13.4\mathbf{j})\ \text{m s}^{-1}$. Find T and the position vector of P relative to O at that time.

(b) Find the time when P is moving perpendicular to its initial direction of motion, and its height above O then.

(c) Find the time when P is moving at $45°$ below the horizontal.

Hint The horizontal component of the velocity stays 12. Write the velocity you want with that $\mathbf{i}$-component, then use the $\mathbf{j}$-component to find t.

B3 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle P is projected from a point O with velocity $(9\mathbf{i} + 12\mathbf{j})\ \text{m s}^{-1}$.

(a) At time T seconds the velocity of P is $(9\mathbf{i} - 7.6\mathbf{j})\ \text{m s}^{-1}$. Find T and the position vector of P relative to O at that time.

(b) Find the time when P is moving perpendicular to its initial direction of motion, and its height above O then.

(c) Find the time when P is moving at $45°$ below the horizontal.

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Answer .....

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EXAMPLE 4

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is projected from a point O on horizontal ground with velocity $(3u\mathbf{i} + 4u\mathbf{j})\ \mathrm{m\ s^{-1}}$, where u is a positive constant. It lands on the ground $120\ \mathrm{m}$ from O.

(a) Show that $u = 7$.

(b) Find the greatest height of the ball.

(c) Find the length of time for which the ball is at least $30\ \mathrm{m}$ above the ground.

Hint Write $\mathbf{r}$ with u in it. The landing time comes from the $\mathbf{j}$-component; the range from the $\mathbf{i}$-component at that time gives an equation in u.

B4 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is projected from a point O on horizontal ground with velocity $(k\mathbf{i} + 2k\mathbf{j})\ \mathrm{m\ s^{-1}}$, where k is a positive constant. It lands on the ground $20\ \mathrm{m}$ from O.

(a) Show that $k = 7$.

(b) Find the greatest height of the ball.

(c) Find the length of time for which the ball is at least $7.5\ \mathrm{m}$ above the ground.

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Answer .....

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EXAMPLE 5

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle is projected from a point O on horizontal ground with velocity $(a\mathbf{i} + b\mathbf{j})\ \text{m s}^{-1}$, where a and b are positive constants. The position of the particle relative to O at time t seconds is $(x\mathbf{i} + y\mathbf{j})\ \text{m}$.

(a) Show that $y = \dfrac{bx}{a} - \dfrac{4.9x^2}{a^2}$.

(b) Given that $a = 14$ and that the particle passes through the point with position vector $(28\mathbf{i} + 8.4\mathbf{j})\ \text{m}$, find b.

(c) Find the distance from O of the point where the particle lands.

Hint x and y are the two components of $\mathbf{r}$. Make t the subject of the x equation and put it into the y equation.

B5 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle is projected from a point O on horizontal ground with velocity $(p\mathbf{i} + q\mathbf{j})\ \text{m s}^{-1}$, where p and q are positive constants. It lands on the ground $20\ \text{m}$ from O, and on the way it passes through the point with position vector $(10\mathbf{i} + 5\mathbf{j})\ \text{m}$ relative to O.

(a) Show that the path has equation $y = \dfrac{qx}{p} - \dfrac{4.9x^2}{p^2}$.

(b) Find p and q.

(c) Find the speed of projection.

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Answer .....

C Practice

A-Level

C1 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is projected from a point O on horizontal ground with velocity $(5\mathbf{i} + 9.8\mathbf{j})\ \mathrm{m\ s^{-1}}$. Tom writes $\mathbf{r} = t(5\mathbf{i} + 9.8\mathbf{j}) + \frac{1}{2}t^2(9.8\mathbf{j})$, finds that the $\mathbf{j}$-component $9.8t + 4.9t^2$ is never 0 for $t > 0$, and says the ball never lands. Explain Tom's mistake and find where the ball lands. [5 marks]

Answer .....

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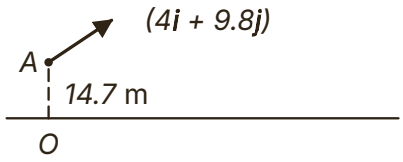
C2 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A stone is projected from a point A which is $14.7\ \text{m}$ vertically above a point O on horizontal ground. Its velocity of projection is $(4\mathbf{i} + 9.8\mathbf{j})\ \text{m s}^{-1}$. Priya writes $\mathbf{r} = t(4\mathbf{i} + 9.8\mathbf{j}) - 4.9t^2\mathbf{j}$, solves $9.8t - 4.9t^2 = 0$ to get $t = 2$, and says the stone lands $8\ \text{m}$ from O. Explain Priya's mistake and find where the stone lands. [5 marks]



Answer .....

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C3 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle is projected with velocity $(6\mathbf{i} + 8\mathbf{j})\ \mathrm{m\ s^{-1}}$. Lee says it is moving perpendicular to its initial direction when its velocity is $(6\mathbf{i} - 8\mathbf{j})\ \mathrm{m\ s^{-1}}$, which happens when $t = \frac{16}{9.8} = 1.63\ \mathrm{s}$. Explain Lee's mistake, and find the correct time and the speed of the particle then. [5 marks]

Answer .....

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C4 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is projected from a point O with velocity $(10\mathbf{i} + 6\mathbf{j})\ \mathrm{m\ s^{-1}}$. Sam finds the position vector $(20\mathbf{i} - 7.6\mathbf{j})\ \mathrm{m}$ when $t = 2$ and says the ball is then moving at $\tan^{-1}\frac{7.6}{20} = 20.8°$ below the horizontal. Explain Sam's mistake, and find the speed of the ball and its direction of motion when $t = 2$. [5 marks]

Answer .....

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C5 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A football is kicked from a point O on horizontal ground with velocity $(12\mathbf{i} + 15\mathbf{j})\ \text{m s}^{-1}$. It is modelled as a particle moving freely under gravity. Kai says: "The model says the ball is in the air for $3.06\ \text{s}$ and lands $36.7\ \text{m}$ from O. With air resistance it would land at the same place, just a little later." Check Kai's two values, explain what is wrong with his last sentence, and state one other assumption made in the model. [4 marks]

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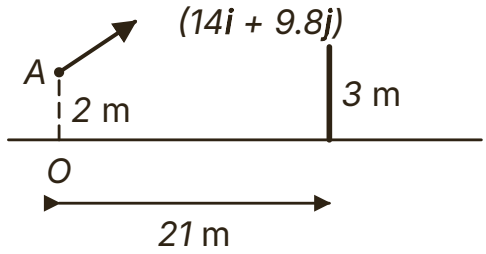
D1 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is thrown from a point A which is $2\ \text{m}$ vertically above a point O on horizontal ground, with velocity $(14\mathbf{i} + 9.8\mathbf{j})\ \text{m s}^{-1}$. A vertical fence of height $3\ \text{m}$ stands on the ground, $21\ \text{m}$ horizontally from O, in the vertical plane of the motion. The ball is modelled as a particle.

(a) Show that the ball passes over the fence, and find by how much it clears the fence. [3 marks]



Answer .....

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(b) Find the speed of the ball as it passes over the fence, and the angle its direction of motion makes with the horizontal at that moment. [4 marks]

Answer .....

(c) Find the distance from O of the point where the ball lands. [3 marks]

Answer .....

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(d) State one reason why the ball may not land at the point found in part (c), and say whether it would land nearer to O or further from O. [2 marks]

Total for D1: 12 marks

D2

> In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is projected from a point O on horizontal ground with velocity $(7\mathbf{i} + 21\mathbf{j})\ \text{m s}^{-1}$. Ben finds the speed of projection, $\sqrt{490} = 22.1\ \text{m s}^{-1}$. He then writes "$22.1 - 9.8t = 0$, so $t = 2.26\ \text{s}$" and gives the greatest height as $\frac{22.1^2}{2\times 9.8} = 25\ \text{m}$.

Ben is wrong.

(a) Explain Ben's mistake, and find the correct greatest height of the ball. [3 marks]

Answer .....

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(b) Find the speed of the ball when it is $10\ \text{m}$ above the ground and moving downwards. [4 marks]

Answer .....

(c) Find the angle between the direction of motion of the ball and the horizontal at that moment. [2 marks]

Answer .....

Total for D2: 9 marks

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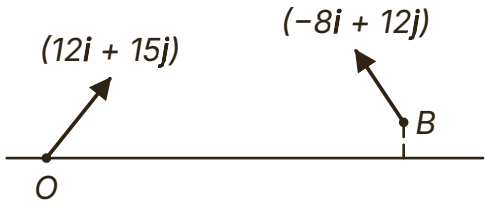
D3 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A particle P is projected from a point O on horizontal ground with velocity $(12\mathbf{i} + 15\mathbf{j})\ \mathrm{m\ s^{-1}}$. At the same instant a particle Q is projected from the point B, with position vector $(30\mathbf{i} + 4.5\mathbf{j})\ \mathrm{m}$ relative to O, with velocity $(-8\mathbf{i} + 12\mathbf{j})\ \mathrm{m\ s^{-1}}$. Both particles move freely under gravity in the same vertical plane.

(a) Show that P and Q collide, and find the time when they collide and the position vector of the point where they collide. [4 marks]



Answer .....

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(b) Find the speed of each particle at the moment of the collision. [4 marks]

Answer .....

(c) Find $\mathbf{r}_P - \mathbf{r}_Q$ in terms of t. Use it to explain why the result of part (a) would be the same for any constant value of g. [4 marks]

Answer .....

Total for D3: 12 marks

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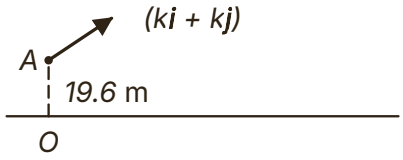
D4 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is thrown from a point A which is $19.6\ \text{m}$ vertically above a point O on horizontal ground. Its velocity of projection is $(k\mathbf{i} + k\mathbf{j})\ \text{m s}^{-1}$, where k is a positive constant. The ball lands on the ground at a point $58.8\ \text{m}$ from O. Find the value of k and the speed of the ball as it lands. [8 marks]



Answer .....

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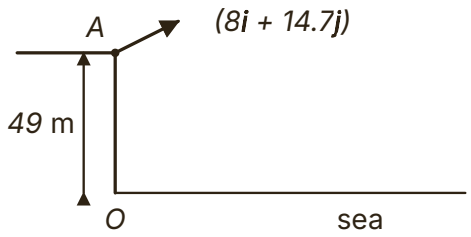
D5 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is projected from a point A which is $49\ \text{m}$ vertically above a point O on the surface of the sea, with velocity $(8\mathbf{i} + 14.7\mathbf{j})\ \text{m s}^{-1}$. At the same instant a boat passes through O moving in the direction of $\mathbf{i}$ with speed $2\ \text{m s}^{-1}$ and constant acceleration $a\mathbf{i}\ \text{m s}^{-2}$. The ball lands on the boat at the point C. The ball and the boat are modelled as particles.

(a) Find the time for which the ball is in the air and the distance OC. [4 marks]



Answer .....

(b) Find the value of a. [3 marks]

Answer .....

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(c) Find the greatest height of the ball above the sea. [3 marks]

Answer .....

(d) State two assumptions made in modelling the motion of the ball. [2 marks]

Total for D5: 12 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

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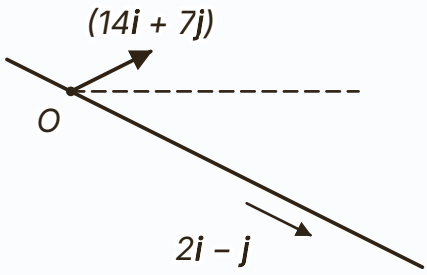
E1 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontal and vertically upwards. A ball is projected from a point O on a hillside with velocity $(14\mathbf{i} + 7\mathbf{j})\ \mathrm{m\ s^{-1}}$. The hillside is modelled as a straight line through O going down in the direction of the vector $(2\mathbf{i} - \mathbf{j})$, in the vertical plane of the motion. The ball lands on the hillside at the point L. Find the distance OL, and the greatest distance of the ball from the hillside, measured perpendicular to the hillside, during its flight. [10 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $\mathbf{r}$ in terms of t. A point on the hillside is a multiple of $(2\mathbf{i} - \mathbf{j})$, so its $\mathbf{j}$-component is minus half its $\mathbf{i}$-component.

Hint 2 The ball is furthest from the hillside when its velocity is parallel to the hillside, not when it is highest.

Hint 3 Find the position at that time. The distance to the hillside is along the direction $(\mathbf{i} + 2\mathbf{j})$, which is perpendicular to $(2\mathbf{i} - \mathbf{j})$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write the velocity of projection as a vector and use the constant acceleration formulae with an acceleration of 9.8 m/s² downwards.	○ ○ ○
I can find the greatest height, the time of flight and the speed and direction of motion at any time.	○ ○ ○
I can use conditions on the velocity or the position to find unknown times and unknown components.	○ ○ ○
I can find the equation of the path and decide whether two moving bodies meet.	○ ○ ○

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