Vectors in kinematics

145 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-year-2-vectors-in-kinematics/

Date

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A Do Now

Quick questions to start. Some check what you need today; some set a slip that costs marks.

A1 Needed today [3 marks]

A particle starts at the point with position vector $(3\mathbf{i} - 2\mathbf{j})$ m and moves with constant velocity $(-2\mathbf{i} + 5\mathbf{j})$ m s^{-1}. Find its position vector after 4 seconds and its speed.

Answer .....

A2 Needed today [3 marks]

A boat has velocity $(-5\mathbf{i} - 12\mathbf{j})$ km h^{-1}, where $\mathbf{i}$ is east and $\mathbf{j}$ is north. Find its speed and the bearing on which it moves.

Answer .....

A3 Needed today [2 marks]

The square of the distance between two ships at time t hours is $D^2 = 5t^2 - 30t + 61$. Find the least value of D and the time at which it happens.

Answer .....

A4 Common slip [2 marks]

A particle has position vector $\mathbf{r} = (t^2 - 9)\mathbf{i} + (4t - 6)\mathbf{j}$ at time t seconds, $t \geq 0$. Ali says that the particle is due north of O when the $\mathbf{j}$-component is zero, so when $t = 1.5$. Find the correct time.

Answer .....

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A5 Common slip [3 marks]

A particle has initial velocity $(3\mathbf{i} + 4\mathbf{j})\ \text{m s}^{-1}$ and constant acceleration $(-\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-2}$. Sam says its speed after 2 seconds is $5 + 2\sqrt{5} = 9.47\ \text{m s}^{-1}$. Find the correct speed.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

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EXAMPLE 1

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle P starts at the point with position vector $(4\mathbf{i} - 10\mathbf{j})$ m with velocity $(-\mathbf{i} + 3\mathbf{j})$ m s^{-1} and moves with constant acceleration $(2\mathbf{i} + 2\mathbf{j})$ m s^{-2}.

(a) Find the speed of P and the bearing on which it moves when $t = 3$.

(b) Find the position vector of P at time t seconds.

(c) Find the time when P is due east of O, and its distance from O then.

Hint Write each formula as a vector equation, then collect the $\mathbf{i}$ and $\mathbf{j}$ parts. A bearing is measured clockwise from north, the direction of $\mathbf{j}$.

B1 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle Q starts at the point with position vector $(-6\mathbf{i} + 12\mathbf{j})$ m with velocity $(\mathbf{i} + \mathbf{j})$ m s^{-1} and moves with constant acceleration $(2\mathbf{i} - 2\mathbf{j})$ m s^{-2}.

(a) Find the speed of Q and the bearing on which it moves when $t = 4$.

(b) Find the position vector of Q at time t seconds.

(c) Find the time when Q is due north of O, and its distance from O then.

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Answer .....

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EXAMPLE 2

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. At noon a ship is at a fixed point O and has velocity $(3\mathbf{i} - 2\mathbf{j})\ \text{km h}^{-1}$. At $16{:}00$ its velocity is $(-5\mathbf{i} + 6\mathbf{j})\ \text{km h}^{-1}$. The ship is modelled as a particle moving with constant acceleration. Its position vector relative to O at time t hours after noon is $\mathbf{s}$ km.

(a) Find the acceleration of the ship.

(b) Find $\mathbf{s}$ in terms of t.

(c) Find the time when the ship is directly north-east of O, and its distance from O then.

Hint Use the two velocities in $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ to find $\mathbf{a}$. North-east means equal components, and both must be positive.

B2 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. At time $t = 0$ a particle is at a fixed point O with velocity $(2\mathbf{i} - 5\mathbf{j})\ \text{m s}^{-1}$. When $t = 6$ its velocity is $(-4\mathbf{i} + \mathbf{j})\ \text{m s}^{-1}$. It moves with constant acceleration.

(a) Find its acceleration and its position vector relative to O at time t seconds.

(b) Find the time when the particle is directly south-west of O, and its distance from O then.

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Answer .....

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EXAMPLE 3

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle P has initial velocity $(6\mathbf{i} - 2\mathbf{j})\ \text{m s}^{-1}$ and constant acceleration $(-2\mathbf{i} + 3\mathbf{j})\ \text{m s}^{-2}$.

(a) Find the time when P is moving due north, and its speed then.

(b) Find the time when P is moving in the direction of the vector $(-\mathbf{i} + 2\mathbf{j})$.

(c) A second particle Q has initial velocity $(2\mathbf{i} + 7\mathbf{j})\ \text{m s}^{-1}$ and constant acceleration $(k\mathbf{i} - 2\mathbf{j})\ \text{m s}^{-2}$. When $t = 5$, Q is moving in the direction of $(\mathbf{i} - \mathbf{j})$. Find k and the speed of Q then.

Hint Write the velocity at time t first. Due north means the $\mathbf{i}$-component is zero; in the direction of a vector means a positive multiple of it.

B3 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle P has initial velocity $(-4\mathbf{i} + 9\mathbf{j})\ \text{m s}^{-1}$ and constant acceleration $(2\mathbf{i} - 3\mathbf{j})\ \text{m s}^{-2}$.

(a) Find the time when P is moving due north, and its speed then.

(b) Find the time when P is moving in the direction of $(\mathbf{i} - \mathbf{j})$.

(c) A second particle R has initial velocity $(3\mathbf{i} - 4\mathbf{j})\ \text{m s}^{-1}$ and constant acceleration $(p\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-2}$. When $t = 4$, R is moving in the direction of $(\mathbf{i} + \mathbf{j})$. Find p and the speed of R then.

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Answer .....

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EXAMPLE 4

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Particle A starts at the point with position vector $(-4\mathbf{i} + 10\mathbf{j})$ m with velocity $(5\mathbf{i} - 2\mathbf{j})$ m s^{-1} and constant acceleration $-2\mathbf{i}$ m s^{-2}. At the same time particle B starts at $(8\mathbf{i} - 6\mathbf{j})$ m and moves with constant velocity $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-1}.
(a) Show that A and B collide, and find the time and the position vector of the point where they collide.
(b) A third particle C starts at $(12\mathbf{i} + 9\mathbf{j})$ m at the same time and moves with constant velocity $(-3\mathbf{i} - \mathbf{j})$ m s^{-1}. Show that A and C do not collide.

Hint Write both position vectors. Solve one component for t, then test those times in the other component.

B4 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Particle P starts at $(2\mathbf{i} + \mathbf{j})$ m with velocity $(\mathbf{i} + 4\mathbf{j})$ m s^{-1} and constant acceleration $(2\mathbf{i} - 2\mathbf{j})$ m s^{-2}. At the same time particle Q starts at $(14\mathbf{i} + \mathbf{j})$ m and moves with constant velocity $(-3\mathbf{i} + 2\mathbf{j})$ m s^{-1}.
(a) Show that P and Q collide, and find the position vector of the point where they collide.
(b) A third particle R starts at $(10\mathbf{i} + 9\mathbf{j})$ m at the same time with constant velocity $(-\mathbf{i} - \mathbf{j})$ m s^{-1}. Show that P and R do not collide.

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Answer .....

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EXAMPLE 5

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Particles P and Q move with constant velocities. At time t seconds, $\mathbf{p} = (2\mathbf{i} - 3\mathbf{j}) + t(4\mathbf{i} + 2\mathbf{j})$ and $\mathbf{q} = (14\mathbf{i} + 6\mathbf{j}) + t(\mathbf{i} - 2\mathbf{j})$, in metres.

(a) Find $\overrightarrow{PQ}$ in terms of t.

(b) Find the time when Q is due east of P, and the distance PQ then.

(c) Find the least distance between P and Q, and the time when it happens.

Hint $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$. The distance is least when PQ^2, a quadratic in t, is least.

B5 [8 marks]

In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Two ships P and Q move with constant velocities. At time t hours after noon, $\mathbf{p} = (6\mathbf{i} + 7\mathbf{j}) + t(2\mathbf{i} - \mathbf{j})$ and $\mathbf{q} = (-14\mathbf{i} + 2\mathbf{j}) + t(5\mathbf{i} + 3\mathbf{j})$, in km.

(a) Find $\overrightarrow{PQ}$ in terms of t.

(b) Find the time of day when Q is due west of P, and the distance PQ then.

(c) Find the least distance between the ships and the time of day when it happens.

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Answer .....

C Practice

A-Level

C1 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle starts at the point with position vector $(3\mathbf{i} - 8\mathbf{j})$ m with velocity $(2\mathbf{i} + 6\mathbf{j})$ m s^{-1} and moves with constant acceleration $(\mathbf{i} - 2\mathbf{j})$ m s^{-2}. Jo writes $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$, sets the $\mathbf{j}$-component $6t - t^2$ equal to 0, and says the particle is due east of O only when $t = 6$. Explain Jo's mistake, and find every time when the particle is due east of O. [5 marks]

Answer .....

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C2 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle has initial velocity $(2\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-1}$ and constant acceleration $(-2.5\mathbf{i} + \mathbf{j})\ \text{m s}^{-2}$. Kai finds its velocity when $t = 4$, then writes "$\tan^{-1}\frac{6}{8} = 36.9^\circ$, so it moves on a bearing of 037°". Find the speed of the particle when $t = 4$ and the correct bearing. [5 marks]

Answer .....

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C3 Particles A and B have position vectors $\mathbf{r}_A = (t^2 - 5)\mathbf{i} + (3t + 1)\mathbf{j}$ and $\mathbf{r}_B = (2t + 10)\mathbf{i} + (t^2 - 2)\mathbf{j}$, in metres, at time t seconds, $t \geq 0$. Mia solves $t^2 - 5 = 2t + 10$ to get $t = 5$, and says "A and B collide when $t = 5$". Explain why Mia's working does not show a collision. Decide whether they collide, and find the distance between them when $t = 5$. [5 marks]

Answer .....

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C4 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle starts at the point with position vector $5\mathbf{j}$ m with velocity $(6\mathbf{i} - 4\mathbf{j})$ m s^{-1} and moves with constant acceleration $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-2}. Leo is asked for the time when the particle is moving due north. He writes $\mathbf{r} = (6t - t^2)\mathbf{i} + (t^2 - 4t + 5)\mathbf{j}$, sets $6t - t^2 = 0$ and answers $t = 6$. Explain Leo's mistake, find the correct time, and find the position vector of the particle at that time. [5 marks]

Answer .....

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C5 Particles P and Q move with constant velocities. At time t seconds, $\mathbf{p} = (\mathbf{i} + 9\mathbf{j}) + t(3\mathbf{i} - 2\mathbf{j})$ and $\mathbf{q} = (9\mathbf{i} + 3\mathbf{j}) + t(\mathbf{i} + 2\mathbf{j})$, in metres. Ana says: "$\overrightarrow{PQ} = (8 - 2t)\mathbf{i} + (4t - 6)\mathbf{j}$. The particles are closest when the $\mathbf{i}$-component is 0, so when $t = 4$, and the least distance is $10\ \text{m}$." Explain why Ana is wrong, and find the least distance between P and Q and the time when it happens. [6 marks]

Answer .....

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D1 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. At noon a ship S is at a fixed point O and has velocity $(6\mathbf{i} + 2\mathbf{j})\ \text{km h}^{-1}$. At $14{:}00$ its velocity is $(-2\mathbf{i} + 6\mathbf{j})\ \text{km h}^{-1}$. The ship is modelled as a particle moving with constant acceleration. At time t hours after noon its position vector relative to O is $\mathbf{s}$ km.

(a) Find the acceleration of S. [2 marks]

Answer .....

(b) Find $\mathbf{s}$ in terms of t. [2 marks]

Answer .....

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(c) Find the time of day when S is due north of O, and the bearing on which S is moving at that time. [4 marks]

Answer .....

(d) A second ship T moves with constant velocity $(-5\mathbf{i} + 5\mathbf{j})\ \mathrm{km\ h^{-1}}$. At noon T is at the point with position vector $(12\mathbf{i} + 4\mathbf{j})\ \mathrm{km}$. Show that the two ships meet, and find the time and the position vector of the point where they meet. [4 marks]

Answer .....

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(e) Explain why the model is unlikely to give the position of S accurately a long time after noon. [2 marks]

Total for D1: 14 marks

D2 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Distances are in km. Two boats A and B move with constant velocities. At noon A is at the point with position vector $(-9\mathbf{i} + 4\mathbf{j})$ and B is at the point with position vector $(11\mathbf{i} - 6\mathbf{j})$. The velocity of A is $(2\mathbf{i} + 2\mathbf{j})\ \text{km h}^{-1}$. B moves at $10\ \text{km h}^{-1}$ in the direction of the vector $(-3\mathbf{i} + 4\mathbf{j})$. At time t hours after noon, $\overrightarrow{AB} = \mathbf{d}\ \text{km}$.

(a) Show that $\mathbf{d} = (20 - 8t)\mathbf{i} + (6t - 10)\mathbf{j}$. [3 marks]

Answer .....

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(b) Find the time of day when B is due north of A, and the distance between the boats at that time. [3 marks]

Answer .....

(c) Find the least distance between the boats. [3 marks]

Answer .....

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(d) The boats can contact each other by radio when they are no more than $5\ \text{km}$ apart. Find, in minutes, the length of time for which they can contact each other. [3 marks]

Answer .....

Total for D2: 12 marks

D3

Relative to a fixed point O, particle P starts at O at time $t = 0$ and moves with constant velocity $(3\mathbf{i} + 6\mathbf{j})\ \text{m s}^{-1}$. Particle Q is at rest at the point A with position vector $(21\mathbf{i} + 6\mathbf{j})\ \text{m}$. When $t = 1$, Q is released and moves with constant acceleration $(-2\mathbf{i} + 4\mathbf{j})\ \text{m s}^{-2}$. Ben writes $\mathbf{r}_Q = (21\mathbf{i} + 6\mathbf{j}) + \frac{1}{2}t^2(-2\mathbf{i} + 4\mathbf{j})$. From the $\mathbf{j}$-components he gets $6t = 6 + 2t^2$, so $t^2 - 3t + 3 = 0$, which has no real roots, and he says "P and Q never collide".

Ben is wrong.

(a) Explain Ben's mistake and write down the correct position vector of Q for $t \geq 1$. [2 marks]

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(b) Show that P and Q do collide, and find the time when they collide. [4 marks]

Answer .....

(c) Find the position vector of the point where they collide, and the speed of Q at that moment. [3 marks]

Answer .....

Total for D3: 9 marks

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D4 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle P moves with constant acceleration. At time $t = 0$ it is at a fixed point O with velocity $(5\mathbf{i} - 8\mathbf{j})\ \mathrm{m\ s^{-1}}$. When $t = 4$, P is at the point with position vector $12\mathbf{i}\ \mathrm{m}$. Find the speed of P at the instant when it is moving in the direction of $\mathbf{i}$, and its distance from O at that instant. [8 marks]

Answer .....

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D5 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. A particle P starts at a fixed point with velocity $(-2\mathbf{i} + 11\mathbf{j})\ \mathrm{m\ s^{-1}}$ and moves with constant acceleration $(2c\mathbf{i} - c\mathbf{j})\ \mathrm{m\ s^{-2}}$, where c is a constant. When $t = 2$ the velocity of P is $b(\mathbf{i} + 2\mathbf{j})\ \mathrm{m\ s^{-1}}$, where b is a constant.

(a) Find the value of c and the value of b. [5 marks]

Answer .....

(b) Find the speed of P when $t = 2$, and the angle between its direction of motion and $\mathbf{i}$. [3 marks]

Answer .....

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(c) Show that the speed of P at time t seconds satisfies $|\mathbf{v}|^2 = 11.25t^2 - 45t + 125$, and hence show that the speed of P is least when $t = 2$. [3 marks]

Answer .....

(d) Find the time, other than $t = 0$, at which the speed of P is equal to its initial speed. [2 marks]

Answer .....

Total for D5: 13 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Two drones fly in horizontal planes, with $\mathbf{i}$ and $\mathbf{j}$ horizontal. Drone A passes over a fixed point O at time $t = 0$ with velocity $(10\mathbf{i} - 20\mathbf{j})\ \mathrm{m\ s^{-1}}$ and moves with constant acceleration $4\mathbf{j}\ \mathrm{m\ s^{-2}}$. Drone B, in a higher horizontal plane, passes over O at time $t = T$ seconds, where $T > 0$, with velocity $(25\mathbf{i} - 15\mathbf{j})\ \mathrm{m\ s^{-1}}$, and moves with constant acceleration $-10\mathbf{j}\ \mathrm{m\ s^{-2}}$. Later, B passes directly above A. Find T, the time when B is directly above A, the horizontal position vector of that point relative to O, and the angle between the directions of motion of the two drones at that instant. [10 marks]

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Answer .....

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $\mathbf{r}_A$ in terms of t. Drone B has been flying for $(t - T)$ seconds, so write $\mathbf{r}_B$ in terms of $(t - T)$.

Hint 2 "Directly above" means the horizontal position vectors are equal. The $\mathbf{i}$-components give t in terms of T.

Hint 3 Put that into the $\mathbf{j}$-components to get an equation in T only; reject $T = 0$. Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for each drone, with B's time $(t - T)$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the constant acceleration formulae with vectors to find a position vector and a velocity at any time.	○ ○ ○
I can find speeds and bearings, and use conditions such as due north of O or moving in the direction of a vector.	○ ○ ○
I can decide whether two particles collide, including when one of them starts later.	○ ○ ○
I can find the least distance between two moving particles.	○ ○ ○

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