Modelling with parametric equations

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-modelling-with-parametric-equations/

Date

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A Do Now

Quick questions to start. Some check what you need today; some are slips that often cost marks.

A1 Needed today [2 marks]

A ball's height is $h = 20t - 5t^2$ metres, t seconds after it is kicked. Find the time it lands and its greatest height.

Answer .....

A2 Needed today [2 marks]

Convert 72 km/h into metres per second.

Answer .....

A3 Last lesson [2 marks]

A particle has position $x = 3t$, $y = 4t$, in metres, at time t seconds. How far is it from the origin when $t = 2$?

Answer .....

A4 Common slip [2 marks]

For the ball in A1, Zoe says the model gives a height of -25 m when $t = 5$, so the ball is 25 m below the ground. Explain why she is wrong.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A ball is hit from a height of 2 m. Its position is $x = 15t$, $y = 2 + 10t - 5t^2$, in metres, t seconds later. Find its greatest height and the horizontal distance it has travelled when it lands.

Hint Greatest height: the vertex of the quadratic in t. Landing: solve y = 0 and keep the positive root.

B1 A ball is thrown from a height of 1.2 m. Its position is $x = 10t$, $y = 1.2 + 8t - 5t^2$, in metres, t seconds later. Find its greatest height, its horizontal distance at that moment, and the horizontal distance it has travelled when it lands. [5 marks]

Answer .....

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EXAMPLE 2

A ball moves on the path $x = 10t$, $y = 10t - 5t^2$. A wall 3 m high stands 12 m away. Find a Cartesian equation of the path and decide whether the ball clears the wall.

Hint Make t the subject of the x equation, put it into y, then find y when x = 12.

B2 A ball moves on the path $x = 8t$, $y = 12t - 5t^2$, in metres. A fence 2.5 m high stands 16 m away. Find a Cartesian equation of the path, and find by how much the ball clears the fence. [4 marks]

Answer .....

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EXAMPLE 3

A seat on a big wheel has position $x = 10\sin\left(\frac{\pi t}{10}\right)$, $y = 12 - 10\cos\left(\frac{\pi t}{10}\right)$, in metres, t minutes after the ride starts. Find the time for one turn, the lowest and highest heights, and the first time the seat is 17 m high.

Hint One turn is when the angle πt/10 reaches 2π. Then solve 12 − 10cos(πt/10) = 17.

B3 A seat on a big wheel has position $x = 18\sin\left(\frac{\pi t}{12}\right)$, $y = 20 - 18\cos\left(\frac{\pi t}{12}\right)$, in metres, t minutes after the ride starts. Find the time for one turn, the lowest and highest heights, and the first time the seat is 29 m high. [5 marks]

Answer .....

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EXAMPLE 4

Ship A has position $x = 3t$, $y = 1 + 2t$ and ship B has position $x = 10 - 2t$, $y = 2t$, in km, t hours after noon. Decide whether they collide, and find where their paths cross.

Hint A collision needs the same point at the same time. A crossing only needs the same point.

B4 [4 marks]

Ship A has position $x = 2t$, $y = 5 + t$ and ship B has position $x = 12 - t$, $y = 3t - 3$, in km, t hours after noon. Show that the ships collide, and find the time and the point.

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EXAMPLE 5

A slide is modelled by $x = 2t$, $y = 9 - t^2$, $0 \leq t \leq 3$, in metres. Find a Cartesian equation, the set of values of x, and the end point of the slide.

Hint Substitute t = x/2. The x values come from the ends of the interval of t.

B5 A slide is modelled by $x = 5t$, $y = 20 - t^2$, $0 \leq t \leq 4$, in metres. Find a Cartesian equation, the set of values of x, and the end point of the slide. [4 marks]

Answer .....

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C1 A shell is fired from level ground. Its position is $x = 30t$, $y = 40t - 5t^2$, in metres, t seconds later. Find how far away it lands and its greatest height. [4 marks]

Answer .....

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C2 A ski jumper leaves the top of a slope at O. Her position is $x = 25t$, $y = -5t^2$, in metres, t seconds later. The slope is the line $y = -\frac{x}{2}$. Find how far down the slope, measured along it, she lands. [5 marks]

Answer .....

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C3 A point P on the rim of a wheel of radius 0.3 m has position $x = 0.3(\theta - \sin\theta)$, $y = 0.3(1 - \cos\theta)$ as the wheel rolls, where θ is the angle turned. Find the greatest height of P and the value of x there, and how far the wheel moves forward in one full turn. [4 marks]

Answer .....

C4 A ball is kicked along level ground. Its position is $x = 18t$, $y = 15t - 4.9t^2$, in metres. Ana works out the range using $5t^2$ in place of $4.9t^2$ and gets 54 m. Find the range from the model, to 3 significant figures. [4 marks]

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C5 A drone flies from a point 60 m above the ground. Its position is $x = 4t$, $y = 60 - 3t$, in metres, t seconds later. Find the time and the horizontal distance when it is 15 m above the ground, the distance it has flown by then, and its speed. [5 marks]

Answer .....

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D Exam-style questions

Edexcel exam style

D1 A golf ball is hit from level ground. Its position is modelled by $x = 30t$, $y = 25t - 5t^2$, in metres, t seconds after it is hit.

(a) Find the horizontal distance the ball travels before it lands. [3 marks]

Answer .....

(b) Find a Cartesian equation of the path of the ball. [3 marks]

Answer .....

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(c) Find the horizontal distances at which the ball is 20 m above the ground. [3 marks]

Answer .....

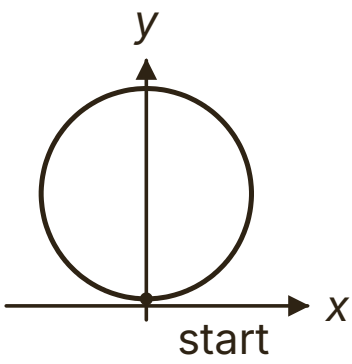
(d) Give two reasons why the model may not give an accurate range. [2 marks]

Total for D1: 11 marks

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D2 The figure shows the path of a seat on a big wheel. Its position is $x = 30\sin\left(\frac{\pi t}{15}\right)$, $y = 32 - 30\cos\left(\frac{\pi t}{15}\right)$, in metres, t minutes after the seat leaves the platform.

(a) Show that the path is a circle, and give its centre and radius. [3 marks]



(b) Find the greatest height of the seat and the time it is first reached. [2 marks]

Answer .....

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(c) Find for how long, in each turn, the seat is more than 47 m above the ground. [4 marks]

Answer .....

Total for D2: 9 marks

D3

> Ravi says: "The ball reaches its greatest height when it has travelled half of its horizontal distance."

Ravi is wrong.

(a) Find the greatest height of the ball and the horizontal distance at that moment. [4 marks]

Answer .....

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(b) Find the horizontal distance the ball travels before it lands. [3 marks]

Answer .....

(c) Explain why Ravi is wrong. [2 marks]

Total for D3: 9 marks

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D4 Two drones fly at the same height. Drone A has position $x = 2t + 1$, $y = 3t$ and drone B has position $x = 3t - 1$, $y = 8t - 5$, in km, t hours after they take off. Find the least distance between the drones, and the time it happens. [7 marks]

Answer .....

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D5 A car drives round a track. Its position is $x = 200\cos\left(\frac{t}{20}\right)$, $y = 150\sin\left(\frac{t}{20}\right)$, in metres, t seconds after it starts.

(a) Find a Cartesian equation of the track. [2 marks]

Answer .....

(b) Find the time for one lap, to 3 significant figures. [2 marks]

Answer .....

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(c) Find the first time the car is on the line $y = x$, and its position then. [4 marks]

Answer .....

Total for D5: 8 marks

D6 A stone is thrown from the top of a cliff 45 m above the sea. Its position is $x = 10t$, $y = 45 + 10t - 5t^2$, in metres, t seconds later.

(a) Find the time the stone hits the sea, and its horizontal distance from the cliff then. Give your answers to 3 significant figures. [4 marks]

Answer .....

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(b) State the domain of t for which the model holds. [2 marks]

Answer .....

(c) Find a Cartesian equation of the path, and give the set of values of x. [3 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A ball is kicked from level ground with speed 20 m/s at an angle α above the horizontal. Its position is $x = 20t\cos\alpha$, $y = 20t\sin\alpha - 5t^2$, in metres. A wall 4 m high stands 20 m away. Find the range of values of α for which the ball passes over the wall. Give the angles to 1 decimal place. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Find the time the ball reaches the wall, then its height there.

Hint 2 Write $\sec^2\alpha$ as $1+\tan^2\alpha$ to get an inequality in $\tan\alpha$.

Hint 3 Solve the quadratic in $\tan\alpha$. The ball clears the wall for values of $\tan\alpha$ between the two roots.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the greatest height and the range of a projectile given by parametric equations.	○ ○ ○
I can find a Cartesian equation for a path and use it to decide whether an object clears an obstacle.	○ ○ ○
I can decide whether two moving objects collide, or only cross paths.	○ ○ ○
I can state a sensible domain for a model and give a reason a model may not be accurate.	○ ○ ○

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