

# Modelling with quadratics

71 marks



Hints online, answers free  
with an account:  
[mathswariz.uk/worksheets/a-level-modelling-with-quadratics/](https://mathswariz.uk/worksheets/a-level-modelling-with-quadratics/)

Date \_\_\_\_\_

## **A** Do Now

Quick questions to start.

- |                                                                                                                                                                                         |                                                                                                                                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>A1</b> <span style="border: 1px solid orange; padding: 2px;">Needed today</span> [2 marks]</p> <p>Write <math>4x^2 - 24x + 29</math> in the form <math>a(x + p)^2 + q</math>.</p> | <p><b>A2</b> <span style="border: 1px solid orange; padding: 2px;">Needed today</span> [2 marks]</p> <p>Solve <math>5x^2 - 3x - 4 = 0</math>.<br/>Give your answers to 3 significant figures.</p> |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Answer .....

Answer  $x = \dots\dots$  or  $x = \dots\dots$

- |                                                                                                                                                                                                                           |                                                                                                                                                               |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>A3</b> <span style="border: 1px solid gray; padding: 2px;">Last lesson</span> [2 marks]</p> <p>Find the range of values of <math>k</math> for which <math>x^2 - 10x + k = 0</math> has two distinct real roots.</p> | <p><b>A4</b> <span style="border: 1px solid gray; padding: 2px;">From chapter 1</span> [2 marks]</p> <p>Expand and simplify <math>(t - 4)(3 - 2t)</math>.</p> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|

Answer .....

Answer  $k \dots\dots$

Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

A stone is thrown from the top of a cliff. Its height above the sea,  $h$  m, after  $t$  seconds is modelled by  $h = 32 + 12t - 5t^2$ ,  $t \geq 0$ . Explain what the 32 means, and find when the stone hits the sea.

**Hint** The 32 is the height when  $t = 0$ . The stone hits the sea when  $h = 0$ . Only a positive time makes sense.

**B1** A ball is thrown from a window. Its height above the ground,  $h$  m, after  $t$  seconds is modelled by  $h = 6.5 + 9t - 4.9t^2$ ,  $t \geq 0$ . Explain what the 6.5 means, and find when the ball hits the ground, to 3 s.f. [3 marks]

Answer  $t = \dots\dots$

**EXAMPLE 2**

The weekly cost,  $\pounds C$ , of running a machine that makes  $x$  hundred items is modelled by  $C = ax^2 + bx + c$ . When  $x = 1$ ,  $C = 520$ ; when  $x = 2$ ,  $C = 480$ ; when  $x = 3$ ,  $C = 500$ . Find  $a$ ,  $b$  and  $c$ .

**Hint** Write three equations. Subtract them in pairs to remove  $c$ , then solve the two equations left.

**B2** The depth,  $d$  cm, of water in a tank  $t$  minutes after a tap is opened is modelled by  $d = at^2 + bt + c$ . When  $t = 0$ ,  $d = 45$ ; when  $t = 2$ ,  $d = 29$ ; when  $t = 4$ ,  $d = 21$ . Find  $a$ ,  $b$  and  $c$ . [3 marks]

Answer  $a = \dots\dots$   $b = \dots\dots$   $c = \dots\dots$

**EXAMPLE 3**

A theatre sells 400 tickets when each costs £20. For every £1 off the price, it sells 25 more tickets. The number sold at a price of £ $p$  is  $n = M - 25p$ . Find  $M$ , and find the price that gives the greatest revenue.

**Hint** Put  $p = 20$ ,  $n = 400$  to find  $M$ .  
Revenue is price  $\times$  number sold. Then complete the square.

**B3** [4 marks]

A café sells 150 sandwiches a day at £4 each. For every 10p off the price it sells 15 more. The number sold at £ $p$  each is  $n = M - 150p$ . Find  $M$ , the best price and the greatest revenue.

Answer  $M = \dots\dots$  £  $\dots\dots$  £  $\dots\dots$

**c Practice**

A-Level

- C1** A ball is thrown from a height of 1.44 m. It reaches its greatest height, [5 marks]  
7.2 m, after 1.2 seconds. Its height,  $h$  m, after  $t$  seconds is modelled by  $h = A - B(t - C)^2$ . Find  $A$ ,  $B$  and  $C$ , and find when the ball lands on the ground, to 3 s.f.

Answer  $A = \dots\dots$   $B = \dots\dots$   $C = \dots\dots$   $t = \dots\dots$

- C2** A ball's height,  $h$  m, after  $t$  seconds is modelled by [3 marks]  
 $h = 1.2 + 7t - 4.9t^2$ . Use the discriminant to show that, according to the model, the ball never reaches a height of 4 m.

- C3** The number of visitors to a museum in month  $t$  after it opens is [3 marks]  
modelled by  $V = 2400 + 220t - 10t^2$ . Find the value of  $t$  at which the model predicts no visitors, and comment on using the model for later months.

Answer  $t = \dots\dots$

## **D** Exam-style questions

Edexcel exam style

- D1** Priya writes: The ball hits the ground when  $8t - 5t^2 = 0$ , so  $t(8 - 5t) = 0$  and the ball is in the air for 1.6 seconds.

A ball is thrown upwards from a balcony. Its height above the ground,  $h$  m, after  $t$  seconds is modelled by  $h = 2.25 + 8t - 5t^2$ ,  $t \geq 0$ .

- (a)** Explain the mistake Priya has made. [2 marks]

- (b) Find when the ball hits the ground. Give your answer to 3 significant figures.

[3 marks]

Answer  $t = \dots\dots$

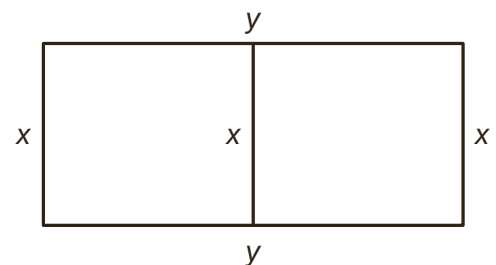
- (c) Write  $h$  in the form  $A - B(t - C)^2$ . Hence find the greatest height of the ball, and for how long the ball is more than 4.25 m above the ground, to 3 s.f. [6 marks]

Answer  $h = \dots\dots \dots$  m  $\dots\dots$  s

Total for D1: 11 marks

- D2** A farmer has 150 m of fencing. She uses all of it to make a rectangular enclosure, split into two pens by one more fence, as in the diagram. Each short side is  $x$  m and each long side is  $y$  m.

- (a) Show that the total area,  $A$  [2 marks]  
 $\text{m}^2$ , of the enclosure is  
 $A = 75x - 1.5x^2$ .



Not drawn accurately

- (b) Write  $A$  in the form  $q - p(x - r)^2$ . Hence find the greatest possible area, and the values of  $x$  and  $y$  that give it. [5 marks]

Answer  $A = \dots\dots$  greatest  $\dots\dots \text{ m}^2$   $x = \dots\dots$   $y = \dots\dots$

- (c) Find the two values of  $x$  for which the area is  $864 \text{ m}^2$ . [3 marks]

Answer  $x = \dots\dots$  or  $x = \dots\dots$

Total for D2: 10 marks

- D3** A company's monthly profit,  $\text{£}P$  thousand, when it makes  $x$  thousand items is modelled by  $P = 36x - 2x^2 - 70$ . Explain what the  $-70$  means. Find, in the form  $a \pm \sqrt{b}$ , the values of  $x$  for which  $P = 0$ . Hence find the least number of items the company must make to make a profit. [5 marks]

Answer  $x = \dots\dots$   $\dots\dots$  items

**D4** A chain hangs between the tops of two posts, 26 m and 14 m high, standing 80 m apart on level ground. Halfway between the posts the chain is 12 m high. At  $x$  m from the taller post the chain is  $h$  m high, where  $h = ax^2 + bx + c$ .

(a) Find the values of  $a$ ,  $b$  and  $c$ .

[4 marks]

Answer  $a = \dots\dots$   $b = \dots\dots$   $c = \dots\dots$

(b) Hence find the least height of the chain above the ground, and how far it is from the taller post.

[4 marks]

Answer  $\dots\dots$  m,  $\dots\dots$  m from the taller post

(c) A lamp hangs from the chain at the point, other than the top of the shorter post, where the chain is 14 m high. How far is the lamp from the taller post?

[2 marks]

Answer  $\dots\dots$  m

Total for D4: 10 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A ball is thrown straight up from the ground. Its height,  $h$  m, after  $t$  seconds is modelled by  $h = ut - 5t^2$ , where  $u$  is a positive constant. The ball passes a window 20 m above the ground twice, 3 seconds apart. Find  $u$ , and the greatest height of the ball. [6 marks]

Answer  $u = \dots\dots \dots$  m

**Extension hints**

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Call the two times  $t_1$  and  $t_2$ . They are the roots of  $5t^2 - ut + 20 = 0$ .
- Hint 2**  $t_1 + t_2 = \frac{u}{5}$  and  $t_1 t_2 = 4$ . Use  $(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1 t_2$ .
- Hint 3**  $\frac{u^2}{25} - 16 = 9$ , so  $u = 25$ . Then complete the square on  $25t - 5t^2$ .

**How did it go?**

Colour one circle on each line: not yet, nearly, yes.

- I can explain a model's numbers. ○ ○ ○
- I can find a model's greatest value. ○ ○ ○
- I can fit a quadratic model to data. ○ ○ ○
- I can say when a model fails. ○ ○ ○