Modelling with series

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-modelling-with-series/

Date

A Do Now

Quick questions to start. They use what you need today.

A1 Needed today [2 marks]

$£2000$ is invested at 3% a year compound interest. Find its value after 5 years.

Answer .....

A2 Needed today [2 marks]

Find the least integer n for which $1.04^n > 2$.

Answer .....

A3 Common slip [2 marks]

Ana earns $£24\,000$ in year 1. Her salary rises by $£800$ each year. Write her salary in year n.

Answer .....

A4 From chapter 3 [2 marks]

A car worth $£12\,000$ loses 15% of its value each year. Write its value after n years.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Sam saves $£50$ in month 1, and 5 more each month than the month before. How much does he save in 24 months?

Hint The amounts go up by the same amount each month, so this is an arithmetic series with $a = 50$, $d = 5$.

B1 Mia walks 30 minutes in week 1 and 4 minutes more each week. Find the total time she walks in the first 20 weeks, and the first week in which she walks more than 100 minutes. [5 marks]

Answer .....

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EXAMPLE 2

$£5000$ is invested at 4% a year. After how many whole years is it first worth more than $£8000$?

Hint Write $5000 \times 1.04^n > 8000$, divide by 5000, then take logs.

B2 A van worth $£18\,000$ loses 12% of its value each year. After how many whole years is it first worth less than $£5000$? [4 marks]

Answer .....

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EXAMPLE 3

Raj pays $£1000$ into an account at the start of each year. It earns 3% a year. Find the value at the end of year 10.

Hint At the end of year 10 the first $£1000$ has grown for 10 years and the last for 1 year. Add the geometric series with first term 1000×1.03 and 10 terms.

B3 Lily pays $£500$ into an account at the start of each year. It earns 5% a year. Find the value at the end of year 8. [5 marks]

Answer .....

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EXAMPLE 4

Job A pays $£30\,000$ in year 1 and rises by $£1500$ a year. Job B pays $£30\,000$ in year 1 and rises by 4% a year. Which pays more in total over 10 years?

Hint Job A is arithmetic and Job B is geometric. Find each sum to 10 terms.

B4 Pay scale P starts at $£20\,000$ and rises by $£1200$ a year. Pay scale Q starts at $£20\,000$ and rises by 5% a year. Find the first year in which Q pays more than P that year, and the total each pays over the first 10 years. [5 marks]

Answer .....

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EXAMPLE 5

A ball is dropped from 10 m. Each bounce reaches 0.6 of the height before. Find the total distance it travels.

Hint It falls 10 m once. After that each height is travelled twice, up and down: $2(6 + 3.6 + \cdots)$.

B5 A ball is dropped from 8 m. Each bounce reaches 0.7 of the height before. Find the total distance it travels, and give one criticism of this model. [5 marks]

Answer .....

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C1 A shop sells 400 units in month 1, and 25 more each month than the month before. Find the total sold in the first 12 months, and the least number of months for the total to pass $30\,000$. [5 marks]

Answer .....

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C2 $£3000$ is invested at 2.5% a year compound interest. Find its value after 5 years, and the least number of whole years for it to be worth more than $£4000$. [5 marks]

Answer .....

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C3 A factory makes 2000 tonnes in year 1. Each year it makes 6% less than the year before. Find the total made in the first 15 years. Show that the total made can never be more than $33\,334$ tonnes. [5 marks]

Answer .....

C4 A population of 400 birds grows by 20% a year. After how many whole years is it first more than 2000? Give one reason the model may not hold for many years. [4 marks]

Answer .....

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D1 Priya runs 5 km on day 1. Each day she runs 0.4 km more than the day before, until she reaches 21 km a day.

(a) How far does she run on day 15? [2 marks]

Answer .....

(b) Find the total distance she runs in the first 15 days. [2 marks]

Answer .....

(c) On which day does she first run 21 km? [2 marks]

Answer .....

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(d) After that she runs 21 km every day. Find the total distance she runs in the first 60 days. [3 marks]

Answer .....

Total for D1: 9 marks

D2

> Lena says: "In month 30 I save $£390$, so in the first 30 months I save $30 \times 390 = £11\,700$."

Lena is wrong.

(a) Lena saves $£100$ in month 1 and $£10$ more each month. Find the correct total for 30 months, and explain her mistake. [4 marks]

Answer .....

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(b) Tom saves $£300$ in month 1 and $£5$ more each month. Find the least number of months after which Lena's total is more than Tom's. [5 marks]

Answer .....

Total for D2: 9 marks

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D3 Town A has $25\,000$ people and its population falls by 2% a year. Town B has $18\,000$ people and its population rises by 3% a year. After how many whole years does Town B first have more people than Town A? [6 marks]

Answer .....

D4 A ball is dropped from 2.5 m. After each bounce it reaches 0.8 of the height before.

(a) Find the height it reaches after the third bounce. [2 marks]

Answer .....

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(b) Find the distance it has travelled when it hits the ground for the tenth time. [4 marks]

Answer .....

(c) In a better model the ball stops once a bounce would reach less than 0.05 m. How many bounces does it make? [3 marks]

Answer .....

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(d) Find the total distance the ball travels in this model, and compare it with the total if it bounced for ever. [3 marks]

Answer .....

Total for D4: 12 marks

D5 Omar pays $£2000$ into an account at the start of each year. It earns 4% a year, added at the end of each year.

(a) Show that the value at the end of year n is $£52\,000(1.04^n - 1)$ [3 marks]

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(b) Find the least n for which the value is more than $£50\,000$. [3 marks]

Answer .....

(c) His sister puts one lump sum into the same account now and pays nothing more. Find how much she must pay in to have the same amount as Omar at the end of year 15. [3 marks]

Answer .....

Total for D5: 9 marks

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D6 A mine produces 8000 tonnes in year 1. Each year it produces 5% less than the year before. It closes after the last year in which it produces at least 3000 tonnes.

(a) Find how many years the mine is open. [4 marks]

Answer .....

(b) Find the total produced while it is open. [3 marks]

Answer .....

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(c) Find the total the model says the mine could ever produce, and what fraction of it is produced while it is open. [2 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A loan of $£10\,000$ is taken out. At the end of each month 1% interest is added, then a repayment of $£R$ is made. The loan is paid off after exactly 24 repayments. Find R. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the amount owed after 1, 2 and 3 months. Look for a geometric series in R.

Hint 2 After n months the amount owed is $10\,000 \times 1.01^n - R(1 + 1.01 + \cdots + 1.01^{n-1})$.

Hint 3 Set the amount owed after 24 months to 0 and sum the series.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can decide whether a situation is arithmetic or geometric and write its terms and sums.	○ ○ ○
I can find the least number of years or months for a total or a term to pass a value, using logs where needed.	○ ○ ○
I can model money paid in at the start of each year, or a loan paid off each month.	○ ○ ○
I can criticise a model and say when it stops being realistic.	○ ○ ○

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