

Modelling with straight lines

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-modelling-with-straight-lines/

Date _____

A Do Now

Quick questions to start.

- | | |
|---|---|
| <p>A1 Needed today [2 marks]</p> <p>Find an equation of the line through $(5, 9)$ and $(15, 33)$ in the form $y = mx + c$.</p> | <p>A2 Last lesson [2 marks]</p> <p>Show that the lines $5x - 2y = 3$ and $4x + 10y = 1$ are perpendicular.</p> |
|---|---|

Answer $y = \dots\dots$

- | | |
|--|--|
| <p>A3 From chapter 3 [2 marks]</p> <p>Solve $12 + 0.8t = 30 - 0.4t$.</p> | <p>A4 GCSE Higher [2 marks]</p> <p>y is directly proportional to x, and $y = 42$ when $x = 12$. Find y when $x = 30$.</p> |
|--|--|

Answer $t = \dots\dots$

Answer $y = \dots\dots$

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Tiles needed, n , is directly proportional to floor area $A \text{ m}^2$, and 4.5 m^2 needs 72 tiles. Find k in $n = kA$, say what k means, and find n for 7.25 m^2 .

Hint A straight line through the origin:

$$k = \frac{n}{A}.$$

B1 Paint used, V litres, is directly proportional to area $A \text{ m}^2$, and 12 m^2 uses 1.8 litres. Find $V = kA$, say what k means, and find the area 4.5 litres covers. [4 marks]

Answer

EXAMPLE 2

A courier charges a fixed fee plus a rate per kg: 4 kg costs £11.20 and 9 kg costs £17.70. Find $C = aw + b$, say what a and b mean, and find the mass of a £25.50 parcel.

Hint The parcels are two points (w, C) on one line.

B2 Hall hire is a fixed fee plus a rate per hour: 3 hours cost £87 and 7 hours £163. Find $C = ah + b$, say what a and b mean, and find the hours in a £353 booking. [5 marks]

Answer

EXAMPLE 3

A stack of n cups is h cm tall:
 $n = 1, 2, 3, 4, 5$ give
 $h = 11.0, 12.4, 13.8, 15.2, 16.6$. Explain
 why a linear model fits, find
 $h = an + b$, and find how many cups
 fit under a 40 cm shelf.

Hint Find the step in h each time n goes up
 by 1.

B3 A stack of n chairs is h [4 marks]
 cm tall: $n = 2, 4, 6, 8$
 give
 $h = 84, 96, 108, 120$.
 Explain why a linear
 model fits, find
 $h = an + b$, and find the
 most chairs in a stack
 under 190 cm.

Answer $h = \dots\dots$ $n = \dots\dots$

c Practice

A-Level

C1 A cyclist starts 6 km from home and rides straight away from home [4 marks]
 at 18 km per hour. (a) Write a model for the distance d km from home
 t minutes later. (b) Find when the cyclist is 15 km from home. (c)
 State one assumption of the model.

Answer $d = \dots\dots$ $t = \dots\dots$

- C2** Data set P is $(0, 3.5), (2, 8.1), (4, 12.7), (6, 17.3)$. Data set Q is $(0, 2), (1, 5), (2, 10), (3, 17)$. Which set fits a linear model? Find its equation. [4 marks]

Answer set $y = \dots\dots$

- C3** The number of visitors n to a café on a day is modelled by $n = 6.5T - 48$, where $T^\circ\text{C}$ is the temperature. (a) Say what 6.5 means. (b) Explain why the model cannot be used when $T = 5$. [3 marks]

- C4** A machine is bought for £14 400 and loses £1650 in value each year. [4 marks]
Write a linear model for its value V after t years, find after how many complete years its value is first below £3000, and give one reason the model may not be realistic.

Answer $V = \dots\dots$ years

D1

A hedge grows at a constant rate. It is 1.35 m tall 2 weeks after it is planted and 1.83 m tall after 8 weeks. Priya says: "It grows 0.08 m a week, so its height is directly proportional to the number of weeks."

Priya is wrong.

- (a) Show that a model for the height h m after w weeks is [3 marks]
 $h = 0.08w + 1.19$.

- (b) Explain why Priya is wrong. [2 marks]

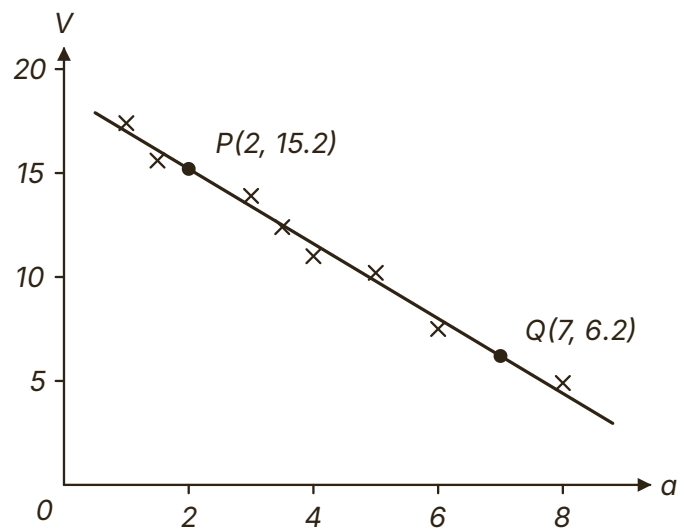
- (c) Hence find the least whole number of weeks after which the [3 marks]
model gives a height over 2 m.

Answer weeks

Total for D1: 8 marks

D2 The scatter graph shows the age a years and the value V thousand pounds of some cars of one model. A line of best fit passes through $P(2, 15.2)$ and $Q(7, 6.2)$.

- (a) Use P and Q to [4 marks]
find a model in the
form $V = ma + c$.
Use it to estimate
the value of a car
that is 4.5 years
old.



Answer $V = \dots\dots \pounds \dots\dots$

- (b) Say what -1.8 and 18.8 mean. [2 marks]

- (c) Hence explain why the model is not reliable for a car 12 years old. [2 marks]

Total for D2: 8 marks

D3 The price $\pounds P$ of a concert ticket and the number Q of tickets are modelled by the demand line $P = 52 - 0.04Q$ and the supply line $P = 0.02Q + 7$. The equilibrium point is where they meet.

(a) Find the price and the number of tickets at the equilibrium point. [3 marks]

Answer $Q = \dots\dots$ $P = \dots\dots$

(b) A booking charge makes the supply line $P = 0.02Q + 10$. Hence [3 marks]
find how many fewer tickets are sold.

Answer $\dots\dots$ fewer

Total for D3: 6 marks

D4 Tank A holds 2400 litres and drains at 35 litres a minute; tank B holds 300 litres and fills at 25 litres a minute, from the same time. Model each volume after t minutes, find when they are equal and that volume, and find the largest t for which the model for A can be used. [6 marks]

Answer

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A phone company has three monthly plans. Plan A: £12 plus 5p a minute. Plan B: £20 plus 2p a minute. Plan C: £35 with no charge per minute. For how many different whole numbers of minutes is plan B the cheapest? [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write each cost in pence for m minutes.
Hint 2 B beats A once $m > 266.7$; B beats C while $m < 750$.
Hint 3 Count the whole numbers from 267 to 749.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write a straight-line model. ☐ ☐ ☐

I can say what a and b mean. ☐ ☐ ☐

I can test whether a linear model fits. ☐ ☐ ☐

I can say when a model stops working. ☐ ☐ ☐