Modelling with trigonometric functions

117 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-modelling-with-trig-functions/

Date

A Do Now

Quick questions to start.

A1 Last lesson [3 marks]

Write $20\cos x + 21\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 2 decimal places.

Answer .....

A2 Today's idea [2 marks]

The depth of water in a harbour is $D = 5 + 3\sin(30t)°$ metres. State the greatest depth and the least depth.

Answer .....

A3 Needed today [2 marks]

t is in hours. State the period of $\sin(30t)°$ and the period of $7 - 2\cos(15t)°$.

Answer .....

A4 From chapter 6 [2 marks]

Solve $\sin(30t)° = 0.5$ for $0 \leq t < 12$.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The height above the ground of a car on a big wheel is $H = 18 - 16\cos(12t)^\circ$ metres, t minutes after the car is at its lowest point. Find the greatest height, the first time it happens, and the time for one turn of the wheel.

Hint −16 cos(12t)° is greatest when cos(12t)° = −1. One turn is when 12t goes up by 360.

B1 [5 marks]

A car on a different wheel has height $H = 20 - 18\cos(9t)^\circ$ metres, t minutes after it is at its lowest point. Find the greatest height and the first time it happens, the time for one turn, and the first time the car is 29 m above the ground.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2

The temperature in a town is modelled by $T = 12 + 3\cos(15t)^\circ + 4\sin(15t)^\circ$ °C, t hours after midnight. Find the greatest temperature and the first time it happens, to the nearest minute.

Hint Write 3 cos(15t)° + 4 sin(15t)° as R cos(15t − α)° first. Then the greatest value of the cosine is 1.

B2 [6 marks]

The temperature in a greenhouse is modelled by $T = 15 + 12\sin(15t)^\circ - 5\cos(15t)^\circ$ °C, t hours after midnight. Write $12\sin(15t)^\circ - 5\cos(15t)^\circ$ in the form $R\sin(15t - \alpha)^\circ$, where $R > 0$ and $0 < \alpha < 90$. Hence find the greatest temperature and the first time it happens, to the nearest minute.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 3

The depth of water at a harbour entrance is $D = 6 + 2.5\sin(30t)^\circ$ metres, t hours after midnight, $0 \leq t < 12$. A boat needs at least 7 m of water. Find the times between which the boat can enter, to the nearest minute.

Hint Solve the equality first: 6 + 2.5 sin(30t)° = 7. The depth is at least 7 m between the two solutions.

B3 The depth of water in a harbour is $D = 7 - 3\cos(30t)^\circ$ metres, t hours after midnight, $0 \leq t \leq 12$. A ship needs at least 8 m of water. Find the times between which the ship can enter, to the nearest minute. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4

On one day high tide is 8.2 m at $03{:}00$ and the next low tide is 1.4 m at $09{:}00$. The depth is modelled by $D = a + b\sin(kt)^\circ$, t hours after midnight. Find a, b and k, and give one reason the model may not work on later days.

Hint a is halfway between the greatest and least depths, b is half the difference, and high to low is half a period.

B4 [5 marks]

The temperature in a greenhouse is modelled by $T = p - q\cos(15t)^\circ$ °C, t hours after midnight, where p and q are positive. The least temperature is 9 °C at midnight and the greatest is 25 °C at noon. Find p and q, find the temperature the model gives at 4 pm, and give one reason the model may not suit a different day.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 The height of a seat on a big wheel is $H = 25 - 22\cos(10t)^\circ$ metres, t minutes after it is at its lowest point. Kim says: "The wheel turns once every 10 minutes, because the number in front of t is 10." Explain Kim's mistake. Find how long, in each turn, the seat is more than 36 m above the ground. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C2 The depth of water in a harbour is $D = 10 + 3\sin(30t)^\circ - 4\cos(30t)^\circ$ metres, t hours after midnight. Write $3\sin(30t)^\circ - 4\cos(30t)^\circ$ in the form $R\sin(30t - \alpha)^\circ$, where $R > 0$ and $0 < \alpha < 90$. Hence find the greatest depth and the first time after midnight it happens, to the nearest minute. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C3 A weight on a spring moves so that its displacement from a fixed point O is $x = 3\cos 2t + 4\sin 2t$ cm, t seconds after it is released. The angle $2t$ is in radians. Find the greatest distance of the weight from O, and the first time, $t > 0$, when $x = -2.5$. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C4 The number of hours of daylight in a city is modelled by $H = 12 - 4\cos\left(\frac{360d}{365}\right)^\circ$, where d is the number of days after 1 January. Find the greatest number of hours of daylight and the value of d where it happens. Find how many days of the year the model gives more than 14 hours of daylight. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C5 On one day high tide is 5.6 m at $01{:}30$ and the next low tide is 1.2 m at $07{:}45$. The depth is modelled by $D = a + b\cos\left(k(t - 1.5)\right)^{\circ}$ metres, t hours after midnight. Find a, b and k, and the depth the model gives at $05{:}00$, to 2 decimal places. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D1 The height of a passenger on a big wheel is modelled by $h = 16 - 7\sin(6t)^\circ - 7\sqrt{3}\cos(6t)^\circ$ metres, t minutes after the ride starts.

(a) Write $7\sin(6t)^\circ + 7\sqrt{3}\cos(6t)^\circ$ in the form $R\sin(6t + \alpha)^\circ$, where $R > 0$ and $0 < \alpha < 90$. [3 marks]

Answer .....

(b) Find the greatest height of the passenger and the first time it happens. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find for how long, in each turn of the wheel, the passenger is more than 23 m above the ground. [3 marks]

Answer .....

(d) Find the exact height of the passenger when the ride starts, and the least height of the passenger. [2 marks]

Answer .....

Total for D1: 11 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D2

> Ravi says: "So the depth is more than 5 m only from midnight until $02{:}13$."

The depth of water in a harbour is modelled by $D = 4 + 2.5\cos(30t)^\circ$ metres, t hours after midnight. Ravi wants the times when the depth is more than 5 m. He solves $4 + 2.5\cos(30t)^\circ = 5$, gets $\cos(30t)^\circ = 0.4$, so $30t = 66.42$ and $t = 2.214$.

(a) Explain why Ravi is wrong, and find all the times between midnight and noon when the depth is more than 5 m, to the nearest minute. [4 marks]

Answer .....

(b) Find the total time between $00{:}00$ and $24{:}00$ when the model gives a depth of more than 5 m. Give your answer in hours and minutes. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Give two reasons why the model may not give the depth well on a later day. [2 marks]

Total for D2: 9 marks

D3 The temperature in an office is modelled by $\theta = 19 + 2\sqrt{3}\cos(15t)^\circ + 2\sin(15t)^\circ$ °C, t hours after 06:00, $0 \leq t < 24$. The air conditioning runs whenever $\theta > 21$. Find the number of hours each day the air conditioning runs, and the time it first switches off. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D4 A buoy moves up and down on the sea. Its height above its rest level is modelled by $y = 0.6\cos 2t - 0.8\sin 2t$ metres, t seconds after a wave passes, $t \geq 0$. The angle $2t$ is in radians.

(a) Write y in the form $R\cos(2t + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give α to 4 decimal places. [3 marks]

Answer .....

(b) Find the greatest height of the buoy above its rest level and the smallest value of t at which it happens. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the values of t in $0 \leq t \leq \pi$ when the buoy is 0.5 m above its rest level. Give your answers to 3 significant figures. [4 marks]

Answer .....

Total for D4: 10 marks

D5 On one day high tide at a port is 9.8 m at $04{:}30$ and the next low tide is 2.2 m at $10{:}30$. The depth is modelled by $D = a + b\cos\left(k(t - 4.5)\right)^{\circ}$ metres, t hours after midnight.

(a) Find a, b and k. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) A boat in the port needs at least 4 m of water. Find the latest time after $04{:}30$ at which it can leave, to the nearest minute. [3 marks]

Answer .....

(c) The next high tide is at $16{:}55$, not at the time the model gives. Use this to find a better value of k, to 3 significant figures. [2 marks]

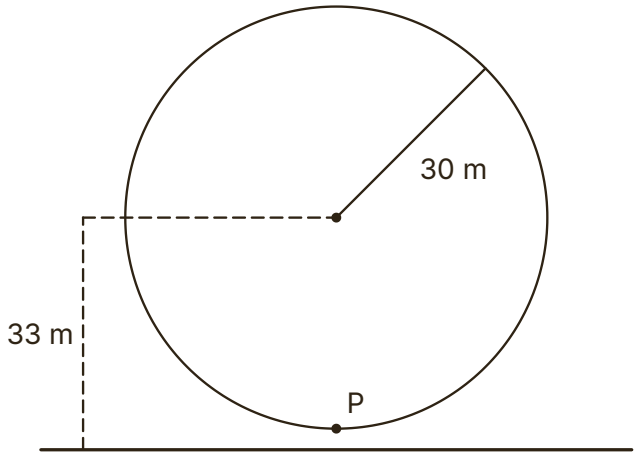
Answer .....

Total for D5: 8 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D6 A big wheel has radius 30 m and its centre is 33 m above the ground. It turns at a steady speed once every 20 minutes. A passenger P is at the lowest point when $t = 0$, where t is in minutes. h m is the height of P above the ground.

(a) Show that $h = 33 - 30\cos(18t)°$. [3 marks]



(b) Find the first time P is 48 m above the ground. Give your answer in minutes and seconds. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) A second passenger Q is at the lowest point when $t = 5$. Find the first time after $t = 5$ when P and Q are at the same height, and that height to 3 significant figures. [4 marks]

Answer .....

Total for D6: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [9 marks]

Two cars are on different big wheels. Their heights above the ground are $h_1 = 20 - 18\cos(12t)^\circ$ metres and $h_2 = 15 - 12\sin(12t)^\circ$ metres, t minutes after they start, $0 \leq t < 30$. Find the times when the two cars are at the same height, and the greatest vertical distance between the two cars. Give your answers to 3 significant figures.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Set $h_1 = h_2$ and collect the cosine and sine terms on one side.

Hint 2 You should get $18\cos(12t)^\circ - 12\sin(12t)^\circ = 5$. Write the left side as $R\cos(12t + \alpha)^\circ$.

Hint 3 $h_1 - h_2 = 5 - R\cos(12t + \alpha)^\circ$. It is furthest from 0 when the cosine is -1.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can read the greatest value, least value and period from a model such as a + b cos(kt)°.	○ ○ ○
I can find every time in an interval when a model takes a given value, and how long it stays above it.	○ ○ ○
I can write a model with a sine and a cosine as one wave to find its greatest value and when it happens.	○ ○ ○
I can find a, b and k from data and give a reason a model may not fit.	○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.