Modulus graphs and combined transformations

118 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-modulus-graphs-and-combined-transformations/

Date

A Do Now

Quick questions to start.

A1 From Year 1 [2 marks]

The point $(3, -4)$ lies on the curve $y = f(x)$.
Write down its image on the curve with equation

(a) $y = f(x + 2)$

(b) $y = 2f(x)$

Answer .....

A2 Lesson 1 [2 marks]

Solve $|2x - 3| = 5$.

Answer .....

A3 From Year 1 [2 marks]

The curve $y = f(x)$ has a maximum point at $(2, 5)$. Write down the coordinates of the maximum point of $y = f(3x)$.

Answer .....

A4 Needed today [3 marks]

$f(x) = x^2 - 4x$. Find the points where $y = f(x)$ meets the x-axis, and the coordinates of its turning point.

Answer .....

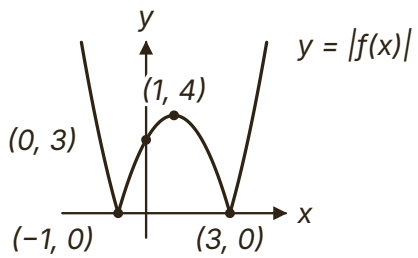
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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$f(x) = x^2 - 2x - 3$. Sketch the graph of $y = |f(x)|$. Give the coordinates of the points where it meets the axes, and of its maximum point.



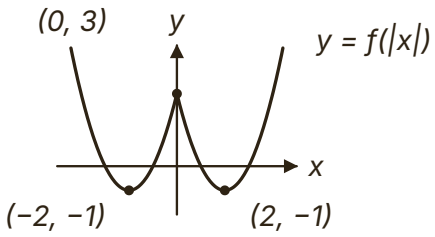
Hint Draw $y = f(x)$ first. Keep the part on or above the x-axis. Reflect the part below the x-axis in the x-axis.

B1 $f(x) = x^2 - 6x + 5$. Sketch the graph of $y = |f(x)|$. Give the coordinates of the points where it meets the axes, and of its maximum point. [4 marks]

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EXAMPLE 2

$f(x) = x^2 - 4x + 3$. Sketch the graph of $y = f(|x|)$. Give the coordinates of the points where it meets the axes, and of its minimum points.



Hint For $x \geq 0$, $f(|x|) = f(x)$. For $x < 0$, $f(|x|) = f(-x)$, so the left half is the reflection of the right half in the y-axis.

B2 $f(x) = x^2 - 6x + 8$. Sketch the graph of $y = f(|x|)$. Give the coordinates of the points where it meets the axes, and of its minimum points. [4 marks]

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EXAMPLE 3

The curve $y = f(x)$ has a turning point at $(1, 4)$. Find the coordinates of the turning point on the curve $y = 2f(x + 3) - 5$.

Hint The changes inside the bracket act on x. The changes outside act on y. Find the x that makes the bracket equal to 1.

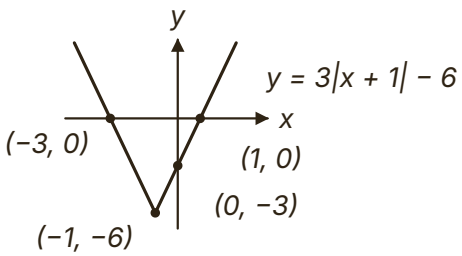
B3 The curve $y = f(x)$ has a turning point at $(4, -6)$. Find the coordinates of the turning point on the curve $y = 3f(2x - 2) + 1$. [3 marks]

Answer .....

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EXAMPLE 4

Sketch the graph of $y = 3|x + 1| - 6$. Give the coordinates of its vertex and of the points where it meets the axes.



Hint $y = |x|$ is translated 1 to the left, stretched by 3 parallel to the y-axis, then translated 6 down.

B4 Sketch the graph of $y = 2|x - 4| - 6$. Give the coordinates of its vertex and of the points where it meets the axes. [4 marks]

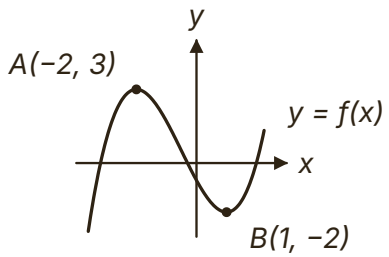
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EXAMPLE 5

The diagram shows the curve $y = f(x)$, with a maximum point $A(-2, 3)$ and a minimum point $B(1, -2)$. Give the coordinates of the images of A and B on

(a) $y = |f(x)|$

(b) $y = f(|x|)$



Hint For $|f(x)|$ only the y-coordinate of a point below the axis changes sign. For $f(|x|)$ only points with $x \geq 0$ are kept, together with their reflections in the y-axis.

B5 The curve $y = f(x)$ has a maximum point $P(-3, 5)$ and a minimum point $Q(2, -4)$. It has no other turning points. Give the coordinates of the turning points of [4 marks]

(a) $y = |f(x)|$ that come from P and Q

(b) $y = f(|x|)$ that come from P and Q

Answer .....

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C1 $f(x) = x - 3$. Raj says: "The graphs of $y = |f(x)|$ and $y = f(|x|)$ are the same." [5 marks]

(a) Work out $|f(x)|$ and $f(|x|)$ when $x = 0$, and when $x = -5$.

(b) Find the values of x for which $|f(x)| = f(|x|)$.

Answer .....

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C2 The point $(4, 1)$ lies on $y = f(x)$. Ella says: "To get $y = f(2x + 6)$, stretch $y = f(x)$ parallel to the x-axis by scale factor $\frac{1}{2}$, then translate it 6 to the left." [4 marks]

(a) Find the image of $(4, 1)$ on $y = f(2x + 6)$.

(b) Find where Ella's two steps send $(4, 1)$, and describe the transformation correctly.

Answer .....

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C3 (a) Sketch the graph of $y = |\ln x|$, $x > 0$. Give the coordinates of the point where it meets the x-axis. [4 marks]

(b) Solve $|\ln x| = 2$.

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C4 Sketch the graph of $y = 5 - 2|x + 1|$. Give the coordinates of its vertex and of the points where it meets the axes, and state the range. [5 marks]

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C5 The curve $y = f(x)$ has turning points $P(-2, 6)$ and $Q(4, -3)$. [4 marks]
Find the coordinates of the images of P and Q on

(a) $y = -2f(x + 1)$

(b) $y = f\left(\frac{1}{2}x\right) - 3$

Answer .....

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C6 $f(x) = (x-1)(x-3)(x+2)$. [5 marks]

(a) Write down the roots of $f(|x|) = 0$ and the y-intercept of $y = f(|x|)$.

(b) The minimum value of $f(x)$ for $x \geq 0$ is about -4.06. How many solutions does $f(|x|) = 2$ have? Give a reason.

Answer .....

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D1 The function f is defined by $f(x) = x^2 - 6x$, $x \in \mathbb{R}$.

(a) Sketch the graph of $y = |f(x)|$. Give the coordinates of the points where it meets the axes, and of its maximum point. [3 marks]

(b) Sketch the graph of $y = f(|x|)$. Give the coordinates of the points where it meets the axes, and of its minimum points. [3 marks]

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(c) Find the coordinates of the turning point of $y = 2f(x - 1) + 5$ [2 marks]

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Answer .....

(d) State the number of solutions of $|f(x)| = k$ when $0 < k < 9$. [2 marks]

Answer .....

Total for D1: 10 marks

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D2

> Ben says: "For any function f, the graphs of $y = |f(x)|$ and $y = f(|x|)$ are both symmetrical about the y-axis."

Ben is wrong.

(a) Show that the graph of $y = f(|x|)$ is always symmetrical about the y-axis. [2 marks]

(b) $f(x) = x^2 - 4x - 5$. Show that the graph of $y = |f(x)|$ is not symmetrical about the y-axis. [2 marks]

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(c) For the same f, sketch the graph of $y = f(|x|)$. Give the coordinates of the points where it meets the axes, and of its minimum points. [4 marks]

Total for D2: 8 marks

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D3 The curve $y = f(x)$ has a maximum point $P(2, 7)$ and passes through $Q(0, 3)$. [9 marks]

Describe fully a sequence of transformations that maps $y = f(x)$ onto $y = 4 - 2f(3x + 6)$. Find the coordinates of the images of P and Q, and state whether the image of P is a maximum or a minimum point.

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D4 The function f is defined by $f(x) = 2|x + 1| - 3$, $x \in \mathbb{R}$.

(a) Sketch the graph of $y = f(x)$. Give the coordinates of its vertex and of the points where it meets the axes. [4 marks]

(b) State the range of f and explain why f has no inverse. [2 marks]

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(c) Give the coordinates of the maximum point of $y = |f(x)|$. [2 marks]

Answer .....

(d) Find the set of values of k for which $|f(x)| = k$ has exactly four solutions. [2 marks]

Answer .....

Total for D4: 10 marks

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D5 The function f is defined by $f(x) = e^x - 4$, $x \in \mathbb{R}$.

(a) Sketch the graph of $y = |f(x)|$. Give the coordinates of the points where it meets the axes, and the equation of its asymptote. [3 marks]

(b) Sketch the graph of $y = f(|x|)$. Give the coordinates of the points where it meets the axes. [3 marks]

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(c) The curve $y = 2f(x - 1)$ meets the x-axis at A and has the horizontal asymptote $y = c$. Find the exact x-coordinate of A and the value of c. [3 marks]

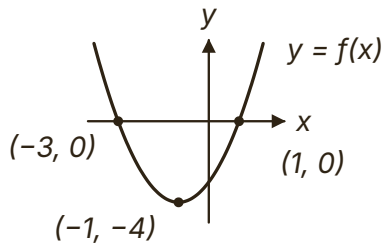
Answer .....

Total for D5: 9 marks

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D6 The diagram shows the curve $y = f(x)$, where $f(x) = x^2 + 2x - 3$. It meets the x-axis at $(-3, 0)$ and $(1, 0)$ and has a minimum point at $(-1, -4)$.

(a) Sketch $y = |f(x)|$. Give the coordinates of its maximum point and of the points where it meets the axes. [3 marks]



(b) Give the coordinates of the minimum point of $y = f(|x|)$ and the points where it meets the x-axis. [2 marks]

Answer .....

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(c) The curve $y = af(x + b)$ has a minimum point at $(2, -12)$. Find the values of a and b. [3 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The curve C has equation $y = |x^2 - 6|x| + 5|$. Find the set of values of k for which the equation $|x^2 - 6|x| + 5| = k$ has exactly eight solutions, and the value of k for which it has exactly three solutions. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Start with $y = x^2 - 6x + 5 = (x - 1)(x - 5)$. Sketch $y = f(|x|)$ first, then take the modulus of that.

Hint 2 For $x \geq 0$ the curve C falls from 5 at $x = 0$ to 0 at $x = 1$, rises to a maximum 4 at $x = 3$, falls to 0 at $x = 5$, then rises. The left half is the reflection in the y-axis.

Hint 3 Count where a horizontal line $y = k$ meets C. Look at k below 4, between 4 and 5, and equal to 5.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch y = |f(x)| by reflecting the parts of y = f(x) below the x-axis.	○ ○ ○
I can sketch y = f(|x|) by keeping the part of y = f(x) for x ≥ 0 and reflecting it in the y-axis.	○ ○ ○
I can find where a point moves under a combined transformation y = af(bx + c) + d.	○ ○ ○
I can describe a combined transformation in the right order.	○ ○ ○

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