

Parallel and perpendicular lines, length and area

73 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-parallel-and-perpendicular-lines/

Date _____

A Do Now

Quick questions to start.

- | | |
|--|--|
| <p>A1 Needed today [2 marks]</p> <p>Find the gradient of a line perpendicular to $2x + 5y - 3 = 0$.</p> | <p>A2 Last lesson [2 marks]</p> <p>Find an equation of the line through $(-2, 7)$ and $(4, -5)$ in the form $y = mx + c$.</p> |
|--|--|

Answer

Answer $y = \dots\dots$

- | | |
|---|---|
| <p>A3 From chapter 1 [2 marks]</p> <p>Write $\sqrt{6^2 + 3^2}$ in the form $a\sqrt{5}$.</p> | <p>A4 From chapter 2 [2 marks]</p> <p>Solve $(x - 3)^2 + 16 = 41$.</p> |
|---|---|

Answer

Answer $x = \dots\dots$ or $x = \dots\dots$

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

For each pair, decide whether the lines are parallel, perpendicular or neither.

(a) $4x - 6y + 5 = 0$ and $y = \frac{2}{3}x - 1$.

(b) $5x + 2y = 7$ and $2x - 5y + 1 = 0$.

Hint Write each line as $y = mx + c$. Parallel: equal gradients. Perpendicular: the gradients multiply to -1 .

B1 The lines l_1 , l_2 and l_3 [3 marks]

have equations

$$3x + 7y - 2 = 0,$$

$$14x - 6y + 9 = 0 \text{ and}$$

$$6x + 14y + 11 = 0.$$

Which two lines are parallel? Which line is perpendicular to the other two?

Answer

EXAMPLE 2

Find an equation of the line through $(2, -3)$ that is perpendicular to $3x - 5y + 4 = 0$. Give it in the form $ax + by + c = 0$, where a , b and c are integers.

Hint Find the gradient of the given line, then use $-\frac{1}{m}$.

B2 Find an equation of the [4 marks]

line through $(-1, 5)$ that is perpendicular to

$$4x + 7y - 2 = 0. \text{ Give it}$$

in the form

$$ax + by + c = 0, \text{ where } a, b \text{ and } c \text{ are integers.}$$

Answer

EXAMPLE 3

The distance between $(2, p)$ and $(p, 8)$ is $2\sqrt{5}$. Find the two possible values of p .

Hint Square the distance:

$(x_2 - x_1)^2 + (y_2 - y_1)^2 = 20$. Then solve the quadratic.

EXAMPLE 4

The points $P(-3, 0)$, $Q(3, 3)$ and $R(1, 7)$ are the vertices of a triangle. Show that angle PQR is a right angle, and find the area of the triangle.

Hint Two sides at Q are perpendicular if their gradients multiply to -1 . Then those two sides are the base and the height.

B3 The distance between $(q, 2)$ and $(4, q)$ is $2\sqrt{13}$. Find the two possible values of q . [4 marks]

Answer $q = \dots\dots$ or $q = \dots\dots$

B4 The points $A(-2, -1)$, $B(2, 1)$ and $C(1, 3)$ are the vertices of a triangle. Show that angle ABC is a right angle, and find the area of the triangle. [4 marks]

Answer

- C1** The points $A(-4, 2)$ and $B(2, 5)$ lie on the line l . The line m passes through B and is perpendicular to l . Find the coordinates of the point where m crosses the x -axis. [4 marks]

Answer (....., 0)

- C2** The lines $(k + 1)x + 3y = 5$ and $2x - 4y + 1 = 0$ are perpendicular. Find the value of the constant k . [3 marks]

Answer $k = \dots\dots$

- C3** The points $A(1, -2)$, $B(7, 1)$, $C(5, 5)$ and $D(-1, 2)$ are the vertices of a quadrilateral. Show that $ABCD$ is a rectangle, and find its area. [5 marks]

D1

Tom is asked for the line through $(5, 1)$ perpendicular to $l : 2x - 3y + 6 = 0$. He writes: the gradient of l is $\frac{2}{3}$, so the perpendicular gradient is $-\frac{2}{3}$; $y - 1 = -\frac{2}{3}(x - 5)$, giving $2x + 3y - 13 = 0$.

Tom is wrong.

- (a) Show that Tom's line is not perpendicular to l , and explain his mistake. [2 marks]

- (b) Find a correct equation of the line in the form $ax + by + c = 0$, where a , b and c are integers. [3 marks]

Answer

- (c) Hence find the shortest distance from the point $(5, 1)$ to the line l . [4 marks]
Give your answer as an exact surd.

Answer

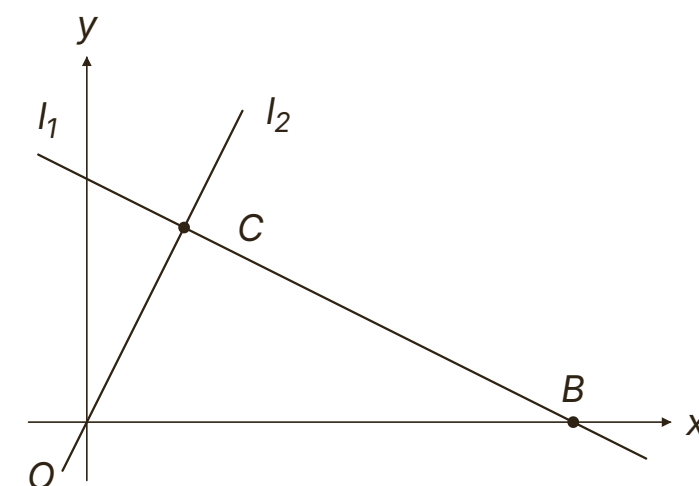
Total for D1: 9 marks

- D2** The line l_1 has equation $x + 2y - 10 = 0$ and crosses the x -axis at B . The line l_2 passes through the origin O and is perpendicular to l_1 . The lines l_1 and l_2 meet at C .

- (a) Find an equation of l_2 . [2 marks]

Answer $y = \dots\dots$

- (b) Find the coordinates of C . [2 marks]



Not drawn accurately

Answer $(\dots\dots, \dots\dots)$

- (c) Find the lengths OC and CB , giving each in the form $k\sqrt{5}$. [3 marks]

Answer

- (d) Hence find the area of triangle OCB . [2 marks]

Answer

Total for D2: 9 marks

D3 The point A has coordinates $(1, 5)$. The points B and C lie on the line $y = x - 2$, and each is a distance $\sqrt{26}$ from A . The x -coordinate of B is less than the x -coordinate of C .

- (a) Show that the x -coordinates of B and C satisfy [3 marks]
 $x^2 - 8x + 12 = 0$.

- (b) Hence find the coordinates of B and C . [2 marks]

Answer $B(\dots, \dots)$ $C(\dots, \dots)$

- (c) Find the exact area of triangle ABC . [4 marks]

Answer

Total for D3: 9 marks

- D4** The lines l_1 and l_2 have equations $3x + 4y - 7 = 0$ and $6x + 8y + 36 = 0$. Show that l_1 and l_2 are parallel, and find the shortest distance between them. [5 marks]

Answer

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The points $A(-1, 2)$ and $C(5, 10)$ are opposite corners of a square $ABCD$. Find B , D and the area of the square. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** The diagonals cross at right angles.
Hint 2 C is 3 across, 4 up from $M(2, 6)$.
Hint 3 B and D are $M \pm (-4, 3)$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can test for parallel lines. ☐ ☐ ☐

I can find a perpendicular line. ☐ ☐ ☐

I can find exact distances. ☐ ☐ ☐

I can find areas on a grid. ☐ ☐ ☐