Parametric differentiation

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-parametric-differentiation/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Differentiate $e^{2t}\cos t$ with respect to t.

Answer .....

A2 Needed today [2 marks]

Find an equation of the line with gradient $-\frac{1}{3}$ that passes through $(2, 5)$. Give it in the form $ax + by + c = 0$, where a, b and c are integers.

Answer .....

A3 From chapter 8 [2 marks]

A curve has parametric equations $x = 2\cos t$, $y = 3\sin t$. Find its Cartesian equation.

Answer .....

A4 Common slip [2 marks]

A curve has $x = t^2$ and $y = t^3$. Write down $\frac{dy}{dt}$ and $\frac{dx}{dt}$, and hence find $\frac{dy}{dx}$ in terms of t.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A curve has parametric equations $x = t^3 - 2t$, $y = t^2 + 4$. Find the gradient of the curve at the point where $t = 2$.

Hint Differentiate x and y with respect to t. Then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$: the y derivative goes on top.

B1 A curve has parametric equations $x = 2t^2 + t$, $y = t^3 - 3t$. Find $\frac{dy}{dx}$ in terms of t, and find the gradient of the curve at the point where $t = 2$. [4 marks]

Answer .....

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EXAMPLE 2

A curve has parametric equations $x = 4\cos t$, $y = 3\sin t$. Find an equation of the normal to the curve at the point where $t = \frac{\pi}{3}$. Give it in the form $ax + by + c = 0$.

Hint Find the point first. The normal has gradient $-\frac{1}{m}$, where m is the gradient of the tangent.

B2 A curve has parametric equations $x = 3\cos t$, $y = 5\sin t$. Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$. Give it in the form $ax + by + c = 0$, where a, b and c are exact. [5 marks]

Answer .....

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EXAMPLE 3

A curve has parametric equations $x = t^2 + 2t$, $y = t^3 - 12t$. Find the coordinates of the points where the tangent to the curve is horizontal.

Hint A horizontal tangent needs $\frac{dy}{dx} = 0$, so $\frac{dy}{dt} = 0$ while $\frac{dx}{dt}$ is not zero.

B3 [5 marks]

A curve has parametric equations $x = t^3 - 3t$, $y = t^2 - 6t$. Find the coordinates of the points where the tangent to the curve is vertical.

Answer .....

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EXAMPLE 4

A curve has parametric equations $x = \ln(t+2)$, $y = \dfrac{1}{t+1}$, $t > -1$. Find an equation of the tangent to the curve at the point where $x = \ln 3$.

Hint You are given x, not t. Solve $\ln(t+2) = \ln 3$ for t first.

B4 [5 marks]

A curve has parametric equations $x = \ln(2t-1)$, $y = \dfrac{t}{t+1}$, $t > \dfrac{1}{2}$. Find an equation of the tangent to the curve at the point where $x = 0$.

Answer .....

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EXAMPLE 5

A curve has parametric equations $x = t^2$, $y = 2t$. The normal to the curve at the point P, where $t = 2$, meets the curve again at Q. Find the coordinates of Q.

Hint Find the normal at P. Then put $x = t^2$ and $y = 2t$ into its equation: you get an equation in t, and you already know one root, $t = 2$.

B5 A curve has parametric equations $x = 2t^2$, $y = 4t$. The normal to the curve at the point where $t = 1$ meets the curve again at Q. Find the coordinates of Q. [6 marks]

Answer .....

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C1 A curve has parametric equations $x = t^2 - 1$, $y = t^3 + t$. Mia finds the tangent at the point where $t = 1$. She writes: "$\frac{dy}{dx} = \frac{2t}{3t^2 + 1} = \frac{1}{2}$." Explain her mistake, and find the correct equation of the tangent. [5 marks]

Answer .....

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C2 A curve has parametric equations $x = 2t + 1$, $y = t^2 - 3t$. Leo is asked for the gradient at the point where $x = 5$. He puts $t = 5$ into $\frac{dy}{dx}$ and gets $\frac{7}{2}$. Explain his mistake and find the correct gradient. [4 marks]

Answer .....

C3 A curve has parametric equations $x = t^3 - 12t$, $y = t^2 - 6t$. To find the stationary point, Zara solves $3t^2 - 12 = 0$. Explain her mistake, and find the coordinates of the stationary point. [4 marks]

Answer .....

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C4 A curve has parametric equations $x = \sin t$, $y = \sin 2t$, $0 \leq t \leq \dfrac{\pi}{2}$. [5 marks]

Find the gradient of the curve at the point where $t = \dfrac{\pi}{3}$, and find the coordinates of the point where the tangent is horizontal.

Answer .....

D Exam-style questions

Edexcel exam style

D1 A curve C has parametric equations $x = t^2 + t$, $y = t^3 - 3t$.

(a) Find $\dfrac{dy}{dx}$ in terms of t. [2 marks]

Answer .....

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(b) Find the coordinates of the points on C where the tangent is horizontal. [3 marks]

Answer .....

(c) Find an equation of the normal to C at the point where $t = 2$. Give it in the form $ax + by + c = 0$, where a, b and c are integers. [4 marks]

Answer .....

Total for D1: 9 marks

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D2

A curve has parametric equations $x = 4\cos t$, $y = \sin 2t$, $0 \leq t < 2\pi$. Sam writes: "$\dfrac{dy}{dx} = \dfrac{2\cos 2t}{-4\sin t}$. The tangent is horizontal when $\sin t = 0$, so the horizontal tangents are at $(4, 0)$ and $(-4, 0)$."

Sam is wrong.

(a) Explain Sam's mistake, and describe the tangent at $(4, 0)$. [2 marks]

(b) Find the coordinates of all the points on the curve where the tangent is horizontal. [5 marks]

Answer .....

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(c) The curve passes through the origin twice. Find the gradient of the curve each time it passes through the origin. [3 marks]

Answer .....

Total for D2: 10 marks

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D3 A curve has parametric equations $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$, $t > 0$. [7 marks]

Find an equation of the tangent to the curve that is parallel to the line $y = 3x$. Give your answer in the form $y = 3x + k$, where k is an exact constant.

Answer .....

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D4 A curve has parametric equations $x = 3t^2$, $y = 6t$. The point P on the curve has parameter $t = p$.

(a) Show that the normal to the curve at P has equation $y + px = 6p + 3p^3$. [4 marks]

(b) Given that $p = 2$, the normal at P meets the curve again at Q. Find the coordinates of Q. [3 marks]

Answer .....

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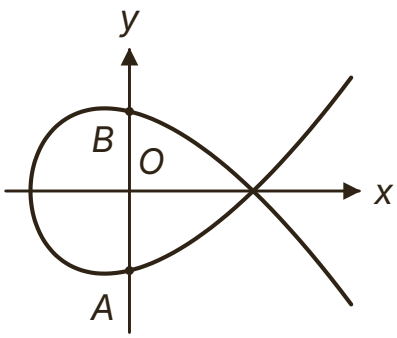
(c) Hence find the area of triangle OPQ, where O is the origin. [3 marks]

Answer .....

Total for D4: 10 marks

D5 The diagram shows the curve with parametric equations $x = t^2 - 4$, $y = t^3 - 9t$. The curve meets the y-axis at A and B.

(a) Find $\frac{dy}{dx}$ in terms of t. [2 marks]



Answer .....

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(b) Find the coordinates of A and B. [2 marks]

Answer .....

(c) Show that the tangents to the curve at A and at B meet on the x-axis, and find the point where they meet. [5 marks]

Answer .....

Total for D5: 9 marks

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D6 A curve has parametric equations $x = e^{2t}$, $y = e^t - t$.

(a) Find $\frac{dy}{dx}$ in terms of t. [2 marks]

Answer .....

(b) Find the coordinates of the stationary point on the curve. [2 marks]

Answer .....

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(c) Find an equation of the tangent to the curve at the point where $t = \ln 2$. Give it in the form $y = mx + c$, where m and c are exact. [4 marks]

Answer .....

Total for D6: 8 marks

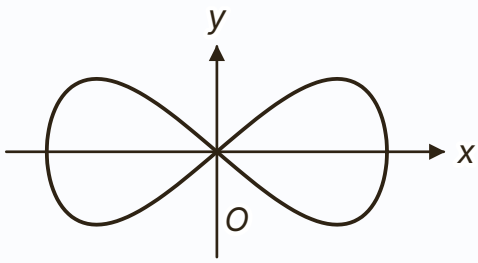
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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The diagram shows the curve with parametric equations $x = 3\cos t$, $y = \sin 2t$, $0 \leq t < 2\pi$. The curve passes through the origin twice. The two tangents to the curve at the origin and the tangent to the curve at the point where $t = 0$ form a triangle. Find the area of this triangle. [9 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 The curve is at the origin when $x = 0$ and $y = 0$. Find the two values of t.

Hint 2 $\dfrac{dy}{dx} = \dfrac{2\cos 2t}{-3\sin t}$. Find its value at each of those values of t.

Hint 3 At $t = 0$, $\dfrac{dx}{dt} = 0$, so that tangent is vertical. Where does it meet the other two?

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find dy/dx as (dy/dt) ÷ (dx/dt) when x and y are given in terms of t.	○ ○ ○
I can find the value of t at a point before working out a gradient there.	○ ○ ○
I can find equations of tangents and normals to a parametric curve.	○ ○ ○
I can find where the tangent is horizontal (dy/dt = 0) or vertical (dx/dt = 0).	○ ○ ○

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