Parametric equations and Cartesian form

119 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-parametric-equations-cartesian-form/

Date

A Do Now

Quick questions to start. Some check what you need today; some are slips that often cost marks.

A1 Needed today [2 marks]

Make t the subject of $x = 3 - 2\ln t$.

Answer .....

A2 Needed today [2 marks]

Write $\cos 2\theta$ in terms of $\sin\theta$ only, and in terms of $\cos\theta$ only.

Answer .....

A3 Common slip [2 marks]

$y = 3 - t^2$ for $-1 < t \leq 2$. Find the range of values of y.

Answer .....

A4 Common slip [2 marks]

Jo writes $\tan^2\theta = \sec^2\theta + 1$. Correct this identity, then find $\tan^2\theta$ when $\sec\theta = 3$.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A curve has parametric equations $x = 2t - 1$, $y = t^2 + 3$. Find a Cartesian equation of the curve.

Hint Make t the subject of the simpler equation, then substitute into the other.

B1 A curve has parametric equations $x = 3t + 2$, $y = 2t^2 - 1$, $-1 \leq t \leq 2$. Find a Cartesian equation of the curve, and state its domain and range. [5 marks]

Answer .....

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EXAMPLE 2

A curve has parametric equations $x = \ln(t+3)$, $y = \dfrac{1}{t+5}$, $t > -2$.

Show that the curve is $y = \dfrac{1}{e^x + 2}$ and state its domain and range.

Hint Undo the ln with e. Then use t > −2 to find the values x and y can take.

B2 [5 marks]

A curve has parametric equations $x = \ln(t-1)$, $y = \dfrac{4}{t+1}$, $t > 2$. Find a Cartesian equation in the form $y = f(x)$, and state the domain and range of f.

Answer .....

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EXAMPLE 3

A curve has parametric equations $x = 3 + 2\cos t$, $y = -1 + 2\sin t$, $0 \leq t < 2\pi$. Show that the curve is a circle and give its centre and radius.

Hint Make cos t and sin t the subjects, then use $\sin^2 t + \cos^2 t = 1$.

B3 [5 marks]

A curve has parametric equations $x = 5\sin t - 2$, $y = 5\cos t + 4$, $0 \leq t \leq \pi$. Find a Cartesian equation of the curve and the set of values of x on it. Describe the curve.

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EXAMPLE 4

A curve has parametric equations $x = \sin t$, $y = \cos 2t$. Find a Cartesian equation and the domain.

Hint Choose the form of cos 2t that uses only sin t.

B4 A curve has parametric equations $x = 2\cos t$, $y = 3\cos 2t + 1$. Find a Cartesian equation in the form $y = f(x)$, and state the domain of f. [4 marks]

Answer .....

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EXAMPLE 5

A curve has parametric equations $x = \sin t$, $y = \sin 2t$, $0 \leq t \leq \frac{\pi}{2}$.

Find a Cartesian equation.

Hint sin 2t = 2 sin t cos t. Find cos t from sin t, and use the interval to choose the sign.

B5 [5 marks]

A curve has parametric equations $x = 2\tan t$, $y = 3\sec t$, $0 \leq t < \frac{\pi}{2}$.

Find a Cartesian equation in the form $y = f(x)$, and state the domain of f.

Answer .....

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C1 A curve has parametric equations $x = t^2 - 1$, $y = 4t$, for all real t. [4 marks]
Max writes "$y = 4\sqrt{x + 1}$". Explain what is wrong, and give a correct Cartesian equation and domain.

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C2 A curve has parametric equations $x = \dfrac{2}{t-1}, y = \dfrac{t+3}{t-1}, t > 1.$ [5 marks]

Show that the curve is part of a straight line, and find the domain and range.

Answer .....

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C3 A curve has parametric equations $x = \cos t + 2$, $y = \cos 2t$. Find a Cartesian equation in the form $y = ax^2 + bx + c$, and state the domain and range. [5 marks]

Answer .....

C4 A curve has parametric equations $x = 2\sec t$, $y = \tan t + 1$. Ana writes "$\frac{x^2}{4} = (y-1)^2 - 1$". Find her mistake and give the correct Cartesian equation in the form $x^2 = f(y)$. [4 marks]

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C5 A curve has parametric equations $x = 3\cot t$, $y = 2\operatorname{cosec} t$, $0 < t < \pi$. Find a Cartesian equation in the form $y = f(x)$. [5 marks]

Answer .....

C6 A curve has parametric equations $x = \tan t$, $y = \cos 2t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$. Show that $y = \frac{1 - x^2}{1 + x^2}$. [4 marks]

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D1 A curve C has parametric equations $x = 3\ln(t+2)$, $y = t^2 - 1$, $t > -1$.

(a) Show that a Cartesian equation of C is $y = e^{\frac{2x}{3}} - 4e^{\frac{x}{3}} + 3$. [3 marks]

(b) State the domain of C. [2 marks]

Answer .....

(c) Find the range of C. [2 marks]

Answer .....

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(d) Find the exact x-coordinate of the point where C meets the x-axis. [2 marks]

Answer .....

Total for D1: 9 marks

D2

> Sam writes: "$x = 1 + \sin t$, so $\sin t = x + 1$ and $y = 1 - 2(x + 1)^2$."

Sam is wrong.

(a) Find the mistake, and give a correct Cartesian equation of C. [3 marks]

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(b) Find the domain and range of C. [3 marks]

Answer .....

(c) Find the coordinates of the points where C meets the line $y = x - 1$. [3 marks]

Answer .....

Total for D2: 9 marks

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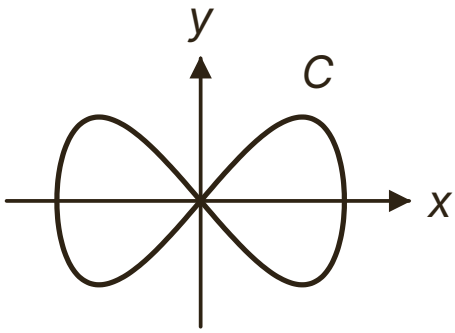
D3 A circle has parametric equations $x = 2 + 5\cos t$, $y = -1 + 5\sin t$, $0 \leq t < 2\pi$. The line $y = x + 2$ meets the circle at two points. By substituting the parametric equations into $y = x + 2$, find the values of t at these points and the coordinates of the points. [7 marks]

Answer .....

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D4 The figure shows the curve C with parametric equations $x = 2\sin t$, $y = \sin 2t$, $0 \leq t < 2\pi$.

(a) Show that a Cartesian equation of C is $4y^2 = x^2(4 - x^2)$. [4 marks]



(b) Find the coordinates of the points where C meets the coordinate axes. [3 marks]

Answer .....

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(c) Find the greatest value of y on C and the value of x where it happens. [2 marks]

Answer .....

Total for D4: 9 marks

D5 A curve C has parametric equations $x = 2 - \frac{1}{t}$, $y = \frac{1}{t^2} + 3$, $t > 0$.

(a) Show that C is part of the curve $y = x^2 - 4x + 7$. [3 marks]

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(b) State the domain of C. [2 marks]

Answer .....

(c) Explain why each horizontal line $y = k$ meets C at most once. [2 marks]

(d) The line $y = 2x + 3$ meets C at P. Find the exact value of t at P. [3 marks]

Answer .....

Total for D5: 10 marks

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D6 An ellipse has parametric equations $x = 3\cos t$, $y = 2\sin t$, $0 \leq t < 2\pi$.

(a) Find a Cartesian equation of the ellipse. [2 marks]

Answer .....

(b) Find the exact coordinates of the point where $t = \frac{\pi}{3}$. [2 marks]

Answer .....

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(c) The line $2x + 3y = 6$ meets the ellipse at two points. Find the values of t at these points and their coordinates. [4 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A curve C has parametric equations $x = \sin t + \cos t$, $y = \sin 2t$, $0 \leq t \leq \pi$. Find a Cartesian equation of C and its domain. Hence find the exact coordinates of every point where C meets the line $y = x$. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Square x. Compare $(\sin t + \cos t)^2$ with $\sin 2t$.

Hint 2 For the domain, write $\sin t + \cos t$ as $\sqrt{2}\sin\left(t + \frac{\pi}{4}\right)$ and look at $t + \frac{\pi}{4}$ for $0 \leq t \leq \pi$.

Hint 3 Solve $x^2 - 1 = x$, then check each root against the domain.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can eliminate the parameter by making t the subject and substituting.	○ ○ ○
I can eliminate the parameter with a trig identity such as cos 2t = 1 − 2 sin² t.	○ ○ ○
I can find the domain and range of a parametric curve from the values of the parameter.	○ ○ ○
I can say which part of a Cartesian curve a parametric curve gives.	○ ○ ○

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