Partial fractions

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-partial-fractions/

Date

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A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Factorise fully $x^3 - x^2 - 6x$.

Answer .....

A2 Last lesson [2 marks]

Write $\frac{3}{x-1} + \frac{2}{x+2}$ as a single fraction.

Answer .....

A3 Needed today [2 marks]

$\frac{5x+1}{(x+1)^2(x-2)}$ is to be split into partial fractions. Write down the form, using constants A, B and C.

Answer .....

A4 Needed today [2 marks]

Solve $A + B = 3$ and $2A - B = 0$.

Answer .....

A5 Needed today [2 marks]

$7x - 1 \equiv A(x-1) + B(x+2)$. By choosing values of x, find A and B.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Express $\dfrac{5x+1}{(x-1)(x+2)}$ in partial fractions.

Hint Write $\dfrac{A}{x-1}+\dfrac{B}{x+2}$, multiply by $(x-1)(x+2)$, then put $x=1$ and $x=-2$.

B1 Express $\dfrac{x+11}{(x+3)(x-1)}$ in partial fractions. [4 marks]

Answer .....

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EXAMPLE 2

Express $\dfrac{x^2+7x-6}{x^3-x^2-2x}$ in partial fractions.

Hint Factorise the bottom fully first: $x(x-2)(x+1)$. Three factors give three fractions.

B2 Express $\dfrac{2x^2+9x-9}{x^3-9x}$ in partial fractions. [5 marks]

Answer .....

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EXAMPLE 3

Express $\dfrac{3x^2 + x + 2}{(x+1)^2(x-3)}$ in partial fractions.

Hint A repeated factor needs two fractions: $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2}$. Substitution gives two constants; compare the x^2 terms for the last one.

B3 Express $\dfrac{x^2 + 5x - 5}{(x-2)^2(x+1)}$ in partial fractions. [5 marks]

Answer .....

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EXAMPLE 4

Express $\dfrac{x+5}{x^2(x+1)}$ in partial fractions.

Hint x^2 is a repeated factor: use $\dfrac{A}{x}+\dfrac{B}{x^2}$. Put $x=0$ and $x=-1$, then compare the x^2 terms.

B4 Express $\dfrac{3x^2-2x+8}{x^2(x-2)}$ in partial fractions. [5 marks]

Answer .....

EXAMPLE 5

Express $\dfrac{8x-1}{(2x-1)(x+1)}$ in partial fractions.

Hint To make $2x-1$ zero, put $x=\dfrac{1}{2}$.

B5 Express $\dfrac{7}{(3x+1)(x-2)}$ in partial fractions. [4 marks]

Answer .....

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C1 Jo writes $\dfrac{5x+1}{(x-1)^2(x+2)} \equiv \dfrac{A}{(x-1)^2} + \dfrac{B}{x+2}$. Show that no constants A and B make this true. Then express the fraction correctly in partial fractions. [5 marks]

Answer .....

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C2 Express $\dfrac{6x^2+12x}{(1-x)(1+2x)^2}$ in partial fractions. [5 marks]

Answer .....

C3 Given that $\dfrac{2x^2+2x+4}{(x-1)^2(x+3)} \equiv \dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+3}$, find A, B and C. [5 marks]

Answer $A = \ldots$, $B = \ldots$, $C = \ldots$

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C4 $f(x) = \dfrac{4x + 10}{(x + 1)(x + 4)}$, $x > -1$. Express $f(x)$ in partial fractions. [5 marks]

Hence show that $f'(x) < 0$ for all $x > -1$.

Answer .....

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D1 Express $\dfrac{7x^2 + 10x}{2x^3 + 3x^2 - 1}$ in partial fractions. [8 marks]

Answer .....

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D2 $f(x) = \dfrac{6 + 5x}{(1 + 2x)(3 - x)}$.

(a) Express $f(x)$ in partial fractions. [3 marks]

Answer .....

(b) Hence find the series expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. Give each coefficient as a simplified fraction. [5 marks]

Answer .....

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(c) State the values of x for which your expansion is valid. [2 marks]

Answer .....

Total for D2: 10 marks

D3

Ben uses cover-up for every term of $\dfrac{9x^2+5x+2}{x^2(x+1)}$. For $\dfrac{A}{x}$ he covers the x of x^2 and puts $x=0$, getting $A=2$. He writes $\dfrac{2}{x}+\dfrac{2}{x^2}+\dfrac{6}{x+1}$.

(a) By substituting $x=1$, show that Ben is wrong. [3 marks]

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(b) Express $\dfrac{9x^2 + 5x + 2}{x^2(x+1)}$ correctly in partial fractions. [5 marks]

Answer .....

Total for D3: 8 marks

D4 $g(x) = x^3 + 2x^2 - 5x - 6.$

(a) Use the factor theorem to factorise $g(x)$ fully. [3 marks]

Answer .....

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(b) Express $\dfrac{3x^2 + 5x + 8}{g(x)}$ in partial fractions. [5 marks]

Answer .....

(c) Hence show that $\dfrac{3x^2 + 5x + 8}{g(x)} > 0$ for all $x > 2$. [3 marks]

Total for D4: 11 marks

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D5 $\dfrac{px+18}{(x-2)(x+4)} \equiv \dfrac{4}{x-2} + \dfrac{q}{x+4}$, where p and q are constants. Call the left side $f(x)$.

(a) Find p and q. [4 marks]

Answer $p = \ldots$, $q = \ldots$

(b) Solve $f(x) = \dfrac{1}{x+4}$. [3 marks]

Answer .....

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(c) Solve $f(x) = 0$. [2 marks]

Answer .....

Total for D5: 9 marks

D6 Given that $\dfrac{2x^2 + kx - 4}{x(x-1)(x+2)} \equiv \dfrac{2}{x} + \dfrac{A}{x-1} + \dfrac{B}{x+2}$ and that $A = 3$, find B and k. [6 marks]

Answer $B = \ldots$, $k = \ldots$

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Show that $\displaystyle\int_1^3 \frac{3x^2+5x+3}{x^2(x+1)}\,dx = 2+\ln 18.$ [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 x^2 is a repeated factor, so use $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$.

Hint 2 You get $\frac{2}{x} + \frac{3}{x^2} + \frac{1}{x+1}$. The middle term integrates to $-\frac{3}{x}$, not to a logarithm.

Hint 3 Put in the limits, then use $\ln a + \ln b = \ln ab$ and $2\ln 3 = \ln 9$ to collect the logarithms.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can split a fraction with distinct linear factors into partial fractions.	○ ○ ○
I can choose the right form when a factor is repeated.	○ ○ ○
I can find the constants by substitution and by comparing coefficients.	○ ○ ○
I can use partial fractions to differentiate, expand as a series or integrate.	○ ○ ○

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