

Pascal's triangle and factorial notation

70 marks



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mathswariz.uk/worksheets/a-level-pascals-triangle-and-factorial-notation/

Date _____

A Do Now

Quick questions to start.

- A1** [Last lesson] [2 marks] Prove that the sum of any two consecutive odd numbers is a multiple of 4.
- A2** [Needed today] [2 marks] Expand and simplify $(3x - 2y)^2$.

Answer

- A3** [From chapter 1] [2 marks] Simplify $(-2x)^3 \times (3x)^{-2}$.
- A4** [Needed today] [2 marks] Expand and simplify $(x + 3)^3$.

Answer

Answer

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Use Pascal's triangle to expand $(3 - 4x)^4$.

Hint Index 4, so use the row 1, 4, 6, 4, 1. Put $(-4x)$ in brackets every time.

- B1** Use Pascal's triangle to expand $(2 - 3x)^5$. [3 marks]

Answer

EXAMPLE 2

In the expansion of $(3 + kx)^4$, the coefficient of x^2 is 150. Find the possible values of k .

Hint The x^2 term is $6 \times 3^2 \times (kx)^2$. Set its coefficient equal to 150.

B2 In the expansion of $(2 - ax)^5$, the coefficient of x^3 is -320 . Find a . [3 marks]

Answer

EXAMPLE 3

Find the coefficient of x^3 in $(2 - 5x)(1 + 2x)^4$.

Hint Expand $(1 + 2x)^4$ first. An x^3 comes from $2 \times$ its x^3 term and $-5x \times$ its x^2 term.

B3 The coefficient of x^2 in $(1 + px)(1 - 2x)^4$ is 0. Find p . [3 marks]

Answer

EXAMPLE 4

Work out $\binom{9}{3}$ without a calculator.

Hence give the 7th entry in the 10th row of Pascal's triangle.

Hint $\binom{9}{3} = \frac{9!}{3!6!}$; cancel the $6!$. The r th entry in the n th row is $\binom{n-1}{r-1}$.

B4 Work out $\binom{11}{4}$ without a calculator. Hence give the 8th entry in the 12th row of Pascal's triangle. [3 marks]

Answer

C1 The 15th row of Pascal's triangle begins 1, 14, 91, ...

(a) Find the next two numbers in the row.

[2 marks]

Answer

(b) Hence find the coefficient of x^3 in the expansion of $(1 - 3x)^{14}$. [2 marks]

Answer

Total for C1: 4 marks

C2 Show that $\frac{(n+1)!}{(n-1)!} = n(n+1)$. Hence solve $\frac{(n+1)!}{(n-1)!} = 132$. [4 marks]

Answer

- C3** $\binom{40}{p} = \frac{40!}{11!q!}$. Find q and the two possible values of p . [3 marks]

Answer

- C4** A fair coin is thrown 12 times. The probability of exactly 4 heads is $\binom{12}{4} \left(\frac{1}{2}\right)^{12}$. Work out this probability, correct to 3 significant figures, and explain why the probability of exactly 8 heads is the same. [3 marks]

Answer

D Exam-style questions

Edexcel exam style

- D1** Sam writes: "The coefficients of $(1 + 2x)^6$ are the 7th row of Pascal's triangle, 1, 6, 15, 20, 15, 6, 1. So the coefficient of x^4 in $(1 + 2x)^6$ is 15."

Sam's answer is wrong.

- (a)** Explain Sam's mistake. [1 mark]

(b) Find the correct coefficient of x^4 in the expansion of $(1 + 2x)^6$. [2 marks]

Answer

(c) The coefficient of x^4 in $(1 + 2x)^6$ is equal to the coefficient of x^2 in $(1 + kx)^6$, where k is a positive constant. Find k . [3 marks]

Answer

Total for D1: 6 marks

D2 In this question y is used for an expression in x .

(a) Use Pascal's triangle to expand $(3 + y)^4$. [2 marks]

Answer

(b) Hence find the coefficient of x^2 in the expansion of $(3 + 2x - x^2)^4$. [3 marks]

Answer

(c) Find the coefficient of x^3 in the expansion of $(3 + 2x - x^2)^4$. [4 marks]

Answer

Total for D2: 9 marks

D3 In this question n is an integer with $n \geq 3$.

(a) Show that $\binom{n}{3} = \frac{n(n-1)(n-2)}{6}$. [2 marks]

(b) Given that $\binom{n}{3} = 4\binom{n}{2}$, find the value of n . [3 marks]

Answer

- (c) Hence, for this value of n , find the value of the constant k for which the coefficient of x^3 in $(1 + kx)^n$ is 7 times the coefficient of x^2 . [3 marks]

Answer

Total for D3: 8 marks

D4 After 4 years, the value, in pounds, of £2000 invested at $r\%$ per year compound interest is $2000 \left(1 + \frac{r}{100}\right)^4$.

- (a) Use Pascal's triangle to show that this is approximately $2000 + 80r + Ar^2$, ignoring higher powers of r , and find the value of the constant A . [3 marks]

Answer

- (b) Hence estimate the value of the investment after 4 years when $r = 3$. [2 marks]

Answer

- (c) Work out the exact value of the investment when $r = 3$, to the nearest penny, and explain why the estimate in part (b) is too small. [3 marks]

Answer

Total for D4: 8 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Prove that $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$ for integers $0 \leq r < n$. [5 marks]

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write both terms with factorials.
Hint 2 Use the common denominator $(r+1)!(n-r)!$.
Hint 3 The numerator is $n!(r+1) + n!(n-r) = (n+1)!$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can expand a bracket to a small power using Pascal's triangle. ☐ ☐ ☐

I can find one coefficient, or an unknown constant from a coefficient. ☐ ☐ ☐

I can work out $n!$ and nCr , and use the r th entry of the n th row. ☐ ☐ ☐

I can solve an equation with nCr or factorials in it. ☐ ☐ ☐