Proving trigonometric identities

115 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-proving-trig-identities/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Write $\cos 2x$ in terms of $\sin x$ only.

Answer .....

A2 From chapter 6 [2 marks]

Show that $\dfrac{\sin^2 x}{1-\cos x} \equiv 1+\cos x$.

A3 Needed today [2 marks]

Show that $\cos(x-45°) \equiv \dfrac{\sqrt{2}}{2}(\cos x + \sin x)$.

A4 Today's idea [3 marks]

Show that $\tan x + \cot x \equiv \dfrac{1}{\sin x \cos x}$.

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Prove that $\dfrac{\sin 2x}{1+\cos 2x} \equiv \tan x$.

Hint Choose the form of cos 2x that makes the 1 disappear: 1 + cos 2x = 2 cos² x.

B1 Prove that $\dfrac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x$. [4 marks]

EXAMPLE 2

Prove that $\cos^4 x - \sin^4 x \equiv \cos 2x$.

Hint It is a difference of two squares.

B2 Prove that $\sin^3 x \cos x - \sin x \cos^3 x \equiv -\dfrac{1}{4}\sin 4x$. [4 marks]

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EXAMPLE 3

Prove that $\dfrac{\sin(A+B)}{\cos A \cos B} \equiv \tan A + \tan B$.

Hint Expand the top, then split the fraction into two.

B3 Prove that $\dfrac{\cos(A-B)}{\sin A \cos B} \equiv \cot A + \tan B$. [4 marks]

EXAMPLE 4

Prove that $\sin 3x \equiv 3\sin x - 4\sin^3 x$.

Hint Write 3x as 2x + x and use the addition formula. Then change every cos² x to 1 – sin² x.

B4 Prove that $\tan 3x \equiv \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$. [5 marks]

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EXAMPLE 5

Prove that $\operatorname{cosec} x - \sin x \equiv \cos x \cot x$.

Hint Write cosec x as 1/sin x and put everything over sin x.

B5 Prove that $\dfrac{1 + \tan^2 x}{1 - \tan^2 x} \equiv \sec 2x$. [4 marks]

C Practice

A-Level

C1 Mo is asked to prove that $\tan x + \cot x \equiv 2\operatorname{cosec} 2x$. He writes $\tan x + \cot x = \dfrac{1}{\sin x \cos x} = \dfrac{1}{\sin 2x} = \operatorname{cosec} 2x$, so he says the identity is wrong. Find his mistake and complete the proof. [4 marks]

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C2 Prove that $\dfrac{\sin 2x - \sin x}{1 + \cos 2x - \cos x} \equiv \tan x.$ [5 marks]

C3 Prove that $\cos(x + y)\cos(x - y) \equiv \cos^2 x - \sin^2 y.$ [4 marks]

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C4

(a) Prove that $8\sin^2 x\cos^2 x \equiv 1 - \cos 4x$. [3 marks]

(b) Hence find the exact value of $\sin^2 \frac{\pi}{8}\cos^2 \frac{\pi}{8}$. [2 marks]

Answer .....

Total for C4: 5 marks

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C5

(a) Prove that $\cot x - \tan x \equiv 2 \cot 2x$. [3 marks]

(b) Hence find the exact value of $\cot \frac{\pi}{8} - \tan \frac{\pi}{8}$. [1 mark]

Answer .....

Total for C5: 4 marks

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D1

(a) Prove that $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x$, for $\cos x + \sin x \neq 0$. [3 marks]

(b) Express $\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3 marks]

Answer .....

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(c) Hence solve $\dfrac{\cos 2x}{\cos x + \sin x} = 1$ for $0° \leq x < 360°$. [3 marks]

Answer .....

(d) State the values of x in $0° \leq x < 360°$ for which $\dfrac{\cos 2x}{\cos x + \sin x}$ is not defined. [2 marks]

Answer .....

Total for D1: 11 marks

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D2

> Jo writes: "$\cos 4x = \cos(2x + 2x) = 2\cos^2 2x - 1$, and $\cos^2 2x = 2\cos^2 x - 1$, so $\cos 4x \equiv 4\cos^2 x - 3$."

Jo says she has proved an identity for $\cos 4x$.

(a) Show that Jo is wrong. [2 marks]

(b) Prove that $\cos 4x \equiv 8\cos^4 x - 8\cos^2 x + 1$. [3 marks]

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(c) Hence solve $8\cos^4 x - 8\cos^2 x + 1 = \frac{1}{2}$ for $0° \leq x < 90°$. [4 marks]

Answer .....

Total for D2: 9 marks

D3 Prove that $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} \equiv 2$, for $\sin x \cos x \neq 0$. [6 marks]

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D4 Let $t = \tan\dfrac{x}{2}$.

(a) Show that $\sin x = \dfrac{2t}{1+t^2}$. [3 marks]

(b) Show that $\cos x = \dfrac{1-t^2}{1+t^2}$. [2 marks]

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(c) Use the substitution to solve $2\sin x - \cos x = 1$ for $0° \leq x < 360°$. Give your answers to 1 decimal place. Check for any solution the substitution cannot find. [5 marks]

Answer .....

Total for D4: 10 marks

D5

(a) Prove that $\cos(x + 60°) + \cos(x - 60°) \equiv \cos x$. [3 marks]

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(b) Prove that $\sin(x + 60^\circ) - \sin(x - 60^\circ) \equiv \sqrt{3}\cos x$. [3 marks]

(c) Hence solve $\cos(x + 60^\circ) + \cos(x - 60^\circ) = \sin(x + 60^\circ) - \sin(x - 60^\circ)$ for $0^\circ \leq x < 360^\circ$. [3 marks]

Answer .....

Total for D5: 9 marks

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D6

(a) Prove that $\sec^2 x + \operatorname{cosec}^2 x \equiv \sec^2 x \operatorname{cosec}^2 x$. [3 marks]

(b) Hence prove that $\sec^2 x + \operatorname{cosec}^2 x \equiv 4\operatorname{cosec}^2 2x$. [3 marks]

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(c) Solve $\sec^2 x + \operatorname{cosec}^2 x = \frac{16}{3}$ for $0 < x < \frac{\pi}{2}$, giving exact answers. [4 marks]

Answer .....

Total for D6: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [8 marks]

Prove that $\cos x \cos 2x \cos 4x \equiv \dfrac{\sin 8x}{8 \sin x}$, for $\sin x \neq 0$. Hence find the exact value of $\cos \dfrac{\pi}{9} \cos \dfrac{2\pi}{9} \cos \dfrac{4\pi}{9}$.

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Multiply the left-hand side by $\sin x$. What does $\sin x \cos x$ make?

Hint 2 Each step doubles the angle: $\sin x \cos x = \frac{1}{2}\sin 2x$, then $\sin 2x \cos 2x = \frac{1}{2}\sin 4x$, and so on.

Hint 3 At $x = \frac{\pi}{9}$ you need $\sin\frac{8\pi}{9}$. The angles $\frac{8\pi}{9}$ and $\frac{\pi}{9}$ add to π.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can start from one side of an identity and work, one line at a time, to the other side.	○ ○ ○
I can choose between the three forms of cos 2x and use sin 2x = 2 sin x cos x in either direction.	○ ○ ○
I can use a proved identity to find an exact value or to solve an equation.	○ ○ ○
I can give a counterexample to show that a statement is not an identity.	○ ○ ○

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