EDEXCEL A-LEVEL MATHEMATICS (9MA0) · PURE PAPERS 1 AND 2 · A-LEVEL PURE A* REVISION · SHEET 1 OF 10

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-a-star-revision-1/

A-Level Pure A* revision 1

108 marks

Date

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X Exam questions

Edexcel 9MA0 Pure style, later parts of long questions. Show all your working: an answer with no method may score no marks. The hints are at the end of the sheet. Use one at a time.

X1 The circle C has equation $x^2 + y^2 - 6x + 4y - 12 = 0$ and centre N. [7 marks]

The tangents to C from the point $P(11, 4)$ touch the circle at the points A and B.

Find the exact area of the quadrilateral $PANB$.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X2 The curve C has parametric equations [8 marks]

$$x = 2t + 1, \qquad y = \frac{4}{t}, \qquad t \neq 0$$

The point P lies on C where $t = 2$. The normal to C at P meets C again at the point Q.

Find the coordinates of Q.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X3 Use the substitution $u = 1 + \sqrt{x}$ to find the exact value of [8 marks]

$$\int_1^4 \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$$

Give your answer in the form $a + \ln b$, where a and b are rational numbers.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X4 The functions f and g are defined by

$$f(x) = \frac{3x+1}{x-2}, \quad x > 2 \qquad\qquad g(x) = x^2 + 2, \quad x \in \mathbb{R}$$

(a) Find $f^{-1}(x)$ and state its domain. [4 marks]

Answer .....

(b) Find the range of gf. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Solve the equation $gf(x) = 18$. [3 marks]

Answer .....

Total for X4: 9 marks

X5 A forestry company plants 400 trees in year 1. In each later year it plants 15% more trees than in the year before.

(a) Show that the total number of trees planted in the first n years is $\frac{8000}{3}(1.15^n - 1)$. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the year in which the total number of trees planted first exceeds $20\,000$. [4 marks]
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

(c) A second company also plants 400 trees in year 1, but in each later year it plants 15% fewer trees than in the year before. [3 marks]
Show that this company can never plant a total of 2700 trees.

Answer .....

Total for X5: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X6 Given that, as $h \to 0$,

$$\frac{\sin 3h}{h} \to 3 \qquad \text{and} \qquad \frac{\cos 3h - 1}{h} \to 0$$

(a) prove, from first principles, that the derivative of $\sin 3x$ is $3\cos 3x$. [5 marks]

Answer .....

(b) Explain how the first of the two given limits is consistent with the small angle approximation for $\sin\theta$. [1 mark]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find an equation of the normal to the curve $y = \sin 3x$ at the point where $x = \dfrac{\pi}{9}$. [3 marks]

Answer .....

Total for X6: 9 marks

X7 The mass, M grams, of yeast in a culture t hours after the start of an experiment is modelled by $M = ab^t$, where a and b are constants. A graph of $\log_{10} M$ against t is a straight line passing through the points $(0, 1.3)$ and $(8, 2.1)$.

(a) Find the values of a and b, giving each to 4 significant figures. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Interpret the values of a and b in the context of the model. [2 marks]

Answer .....

(c) Find the time at which the model predicts a mass of 500 g. Give your answer to 3 significant figures. [2 marks]

Answer .....

(d) Give one reason why the model may not be reliable for large values of t. [1 mark]

Answer .....

Total for X7: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X8 In this question n, p and q are integers.

(a) Prove that if n is not a multiple of 5 then n^2 is not a multiple of 5. [4 marks]

Answer .....

(b) Hence prove by contradiction that $\sqrt{5}$ is irrational. [5 marks]

Answer .....

Total for X8: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X9

$$f(x) = \frac{7 + 7x}{(1 + 3x)(2 - x)}$$

(a) Express $f(x)$ in partial fractions. [3 marks]

Answer .....

(b) Hence find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2.
Give each coefficient as a simplified fraction. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the set of values of x for which the expansion is valid, giving a reason for your answer. [2 marks]

Answer .....

Total for X9: 10 marks

X10 The depth of water, D metres, at the entrance to a harbour t hours after midnight is modelled by

$$D = 5 + 1.2\cos\left(\frac{\pi t}{6}\right) + 0.5\sin\left(\frac{\pi t}{6}\right), \quad 0 \leqslant t < 12$$

(a) Express $1.2\cos\theta + 0.5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3 marks]

Give the exact value of R and the value of α in radians to 4 decimal places.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the greatest depth of water predicted by the model, and the time of day at which it first occurs. Give the time to the nearest minute. [3 marks]

Answer .....

(c) A boat can enter the harbour only when the depth of water is at least 4.35 m. [4 marks]
Show that, between midnight and midday, there are exactly 4 hours during which the boat cannot enter the harbour.

Answer .....

Total for X10: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X11 Water leaks from a hole at the bottom of a tank. At time t minutes the depth of the water is h metres. The rate at which the depth decreases is modelled as being proportional to the square root of the depth.

(a) Write down a differential equation for h in terms of t and a positive constant k. [1 mark]

Answer .....

(b) Initially the depth is 1.44 m, and after 10 minutes the depth is 1 m. Show that $h = (1.2 - 0.02t)^2$. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the time at which the model predicts that the tank is empty. [1 mark]

Answer .....

(d) Explain why the equation $h = (1.2 - 0.02t)^2$ is not a valid model for $t > 60$. [2 marks]

Answer .....

Total for X11: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X12

$$f(x) = e^x - \frac{5}{x-1}, \qquad x \neq 1$$

(a) A student finds that $f(0.9) > 0$ and $f(1.1) < 0$ and concludes that $f(x) = 0$ has a root in the interval $[0.9, 1.1]$. Find the values of $f(0.9)$ and $f(1.1)$ to 2 decimal places, and explain why the student's conclusion is wrong. [3 marks]

Answer .....

(b) Show that $f(x) = 0$ has a root α in the interval $[1.8, 1.9]$. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Show that $f(x) = 0$ can be written as $x = 1 + 5e^{-x}$. Use the iteration $x_{n+1} = 1 + 5e^{-x_n}$ with $x_1 = 1.8$ to find x_2, x_3 and x_4 to 4 decimal places. [3 marks]

Answer .....

(d) By choosing a suitable interval, show that $\alpha = 1.81$ correct to 2 decimal places. [2 marks]

Answer .....

Total for X12: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a complete proof: the assumption, each step with its reason, and a final conclusion.	○	○	○
I can find my own route through a question that has no parts, and give exact answers.	○	○	○
I can set up and solve a model in context, and say what the answer means and where the model stops working.	○	○	○
I can justify a result fully: a range of validity, a nature of a turning point, a root to a stated accuracy.	○	○	○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Complete the square to find the centre and the radius. You do not need the coordinates of A or B.
	Hint 2	What is the angle between a tangent and the radius at the point where they meet?
	Hint 3	$PANB$ is made of two congruent right-angled triangles. Find PN, then PA.
X2	**Hint 1**	Find $\dfrac{dy}{dx}$ in terms of t, then the gradient of the normal at $t = 2$.
	Hint 2	Write the equation of the normal, then replace x and y by their expressions in t.
	Hint 3	You get a quadratic in t. One root is $t = 2$, which is P. The other root gives Q.
X3	**Hint 1**	Make x the subject: $x = (u-1)^2$. Differentiate to replace dx.
	Hint 2	Change the limits to values of u. Then expand the numerator and divide each term by u.
	Hint 3	The answer has $2\ln\dfrac{3}{2}$ in it. Use a log law to write it as a single $\ln$.
X4	**Hint 1**	For (a), the domain of f^{-1} is the range of f. Write $f(x)$ as $3 + \dfrac{7}{x-2}$ and think about $x > 2$.
	Hint 2	For (b), $gf(x)$ means g of $f(x)$. Start from the set of values $f(x)$ can take.
	Hint 3	For (c), you get two values for $f(x)$. Check each one against the range of f before you solve for x.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X5	**Hint 1**	The numbers planted each year are 400, 400×1.15, 400×1.15^2, ... Which kind of series is this?
	Hint 2	For (b), rearrange to $1.15^n > \ldots$ and take logarithms. n must be a whole number of years.
	Hint 3	For (c), the common ratio is now less than 1. What happens to the total as the number of years increases?
X6	**Hint 1**	Start from $\dfrac{f(x+h)-f(x)}{h}$ with $f(x) = \sin 3x$, and expand $\sin(3x+3h)$.
	Hint 2	Group the terms so that one bracket is $\dfrac{\cos 3h - 1}{h}$ and the other is $\dfrac{\sin 3h}{h}$. Keep writing "$\lim$" until you take the limit.
	Hint 3	For (c), use the result of (a) for the gradient of the tangent. The normal is perpendicular to it.
X7	**Hint 1**	Take logarithms of both sides of $M = ab^t$ and compare with $y = mx + c$.
	Hint 2	The intercept and the gradient are logarithms of a and b. You need to undo the logarithm for each.
	Hint 3	For (c), using the straight-line equation avoids rounding errors from a and b.
X8	**Hint 1**	For (a), an integer that is not a multiple of 5 leaves remainder 1, 2, 3 or 4. Writing these as $5k \pm 1$ and $5k \pm 2$ halves the work.
	Hint 2	For (b), start with the word "Assume" and say that the fraction is in its lowest terms. That is the statement you will contradict.
	Hint 3	Part (a) tells you: if p^2 is a multiple of 5, so is p. Use a new letter, $p = 5m$, and show q is a multiple of 5 as well.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

X9	**Hint 1**	For (b), expand each fraction on its own. $(2-x)^{-1}$ is not in the form $(1+\dots)^{-1}$ yet: take out a factor of 2^{-1} first.
	Hint 2	Add the two expansions term by term and check the constant term against $f(0)$.
	Hint 3	For (c), write down the condition for each of the two expansions. The expansion of $f(x)$ needs both of them to be true.
X10	**Hint 1**	Expand $R\cos(\theta-\alpha)$ and compare with $1.2\cos\theta+0.5\sin\theta$ to get $R\cos\alpha$ and $R\sin\alpha$.
	Hint 2	For (b), the greatest value of $\cos(\dots)$ is 1. Set the bracket equal to 0 and solve for t, in hours.
	Hint 3	For (c), you need the values of t for which $\cos(\dots)<-0.5$. Find the two boundary times and subtract.
X11	**Hint 1**	"Decreases" means the rate is negative. "Proportional to the square root" gives $k\sqrt{h}$.
	Hint 2	Put the h terms with dh: integrate $h^{-\frac{1}{2}}$. Use the first condition for the constant of integration and the second for k.
	Hint 3	For (d), work out the value the formula gives at, for example, $t=80$. Does that make sense for a leaking tank?
X12	**Hint 1**	What happens to $\dfrac{5}{x-1}$ as x gets close to 1 from each side?
	Hint 2	A complete sign-change argument has three parts: the two values, the word "continuous", and the conclusion.
	Hint 3	For 2 decimal places, the interval must be $[1.805, 1.815]$: every number in it rounds to 1.81.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.