EDEXCEL A-LEVEL MATHEMATICS (9MA0) · PURE PAPERS 1 AND 2 · A-LEVEL PURE A* REVISION · SHEET 10 OF 10

A-Level Pure A* revision 10

109 marks

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-a-star-revision-10/

Date

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X Exam questions

Edexcel 9MA0 Pure style, later parts of long questions. Show all your working: an answer with no method may score no marks. The hints are at the end of the sheet. Use one at a time.

X1 You may assume that $\sqrt{15}$ is irrational and that $\sqrt{3}$ is irrational.

(a) Prove by contradiction that $\sqrt{3} + \sqrt{5}$ is irrational. [4 marks]

Answer .....

(b) A student says: "The sum of two irrational numbers is always irrational." [3 marks]
Show, with a full explanation, that the student is wrong.

Answer .....

Total for X1: 7 marks

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X2 Show that the equation [7 marks]

$$(\operatorname{cosec} x - \sin x)(\sec x - \cos x) = k$$

has no real solutions when $|k| > \frac{1}{2}$.

Answer .....

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X3 A student is asked to expand $(2+3x)^{-2}$ in ascending powers of x. The student's working begins

$$(2+3x)^{-2} = 2\left(1+\frac{3x}{2}\right)^{-2}$$

(a) Identify the error in this line and write down the correct line. [1 mark]

Answer .....

(b) Find the correct expansion of $(2+3x)^{-2}$ up to and including the term in x^2. [3 marks]

Answer .....

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(c) Hence find the expansion of $\dfrac{1-x}{(2+3x)^2}$ up to and including the term in x^2. [2 marks]

Answer .....

(d) The student says, "Because of the factor $(1-x)$, the expansion in (c) is valid only for $|x|<1$."
State the correct range of validity and explain why the student is wrong. [2 marks]

Answer .....

Total for X3: 8 marks

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X4 The points A, B and C have coordinates $(2, 1, -3)$, $(5, 7, 3)$ and $(p, q, 7)$ respectively, where p and q are constants. The points A, B and C lie on a straight line. [9 marks]

The point D has coordinates $(8, -2, 3)$.

Find the exact area of triangle ACD.

Answer .....

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X5 The function f is defined by $f(x) = 2 - \sqrt{x+3}$, $x \geqslant -3$.
A student writes:

"The range of f is $x \leqslant 2$.
$f^{-1}(x) = (2-x)^2 - 3$, $x \in \mathbb{R}$."

(a) The formula for $f^{-1}(x)$ is correct. Identify the two errors in the student's answer and correct them. [2 marks]

Answer .....

(b) Solve the equation $f^{-1}(x) = 6$. [2 marks]

Answer .....

(c) The function g is defined by $g(x) = x^2 - 3$, $x \in \mathbb{R}$. [2 marks]
Show that $fg(x) = 2 - |x|$.

Answer .....

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(d) Hence solve the equation $fg(x) = \frac{1}{2}x - 4$. [3 marks]

Answer .....

Total for X5: 9 marks

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X6 The curve C has equation $y = 4xe^{-x}$, $x \geqslant 0$. The point A is the maximum point of C, and O is the origin. [9 marks]

Find the exact area of the finite region bounded by C and the straight line OA.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X7 A scientist models the number of bacteria, N, in a dish t hours after the start of an experiment by

$$N = 200e^{0.35t}$$

(a) After 12 hours the scientist counts 9000 bacteria. Find the number predicted by the model at $t = 12$ and comment on the model. [2 marks]

Answer .....

(b) The scientist refines the model to [1 mark]

$$N = \frac{12\,000}{1 + 59e^{-0.4t}}$$

Show that the refined model gives the same number of bacteria at $t = 0$.

Answer .....

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(c) Find the time at which the refined model predicts 9000 bacteria. Give your answer to 3 significant figures. [3 marks]

Answer .....

(d) Compare what the two models predict about the number of bacteria in the long term, and state which is more realistic. [2 marks]

Answer .....

(e) Show that, for small values of t, the refined model gives $N \approx 200e^{0.4t}$. [2 marks]

Answer .....

Total for X7: 10 marks

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X8 The first term of an arithmetic sequence is a and the common difference is d, where $d \neq 0$.

The 1st, 3rd and 11th terms of the arithmetic sequence are the first three terms of a geometric sequence.

(a) Show that $2d = 3a$. [3 marks]

Answer .....

(b) Find the common ratio of the geometric sequence. [2 marks]

Answer .....

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(c) Given also that the sum of the first 10 terms of the arithmetic sequence is 310, find the value of a and the value of d. [3 marks]

Answer .....

(d) Find the sum of the first 6 terms of the geometric sequence. [2 marks]

Answer .....

Total for X8: 10 marks

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X9 The curve C has parametric equations

$$x = t^2 - 4, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

(a) Show that a Cartesian equation of C is $y^2 = x^2(x + 4)$. [3 marks]

Answer .....

(b) The curve crosses itself at one point. Find the coordinates of this point and the gradients of the two tangents to C at this point. [4 marks]

Answer .....

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(c) Find the exact coordinates of the points on C at which the tangent is parallel to the x-axis. [3 marks]

Answer .....

Total for X9: 10 marks

X10

$$f(x) = x^3 - 3kx^2 + 12x, \qquad x \in \mathbb{R}$$

where k is a constant.

(a) Find the set of values of k for which the curve $y = f(x)$ has no stationary points. [4 marks]

Answer .....

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(b) Given that $k = 3$, find the exact x-coordinates of the stationary points and determine the nature of each. [3 marks]

Answer .....

(c) Given that $k = 3$, state the set of values of x for which the curve is concave. [1 mark]

Answer .....

(d) Given instead that $k = 2$, explain why the stationary point of the curve is not a turning point. [2 marks]

Answer .....

Total for X10: 10 marks

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X11 The value of a machine, $£V$ thousand, at time t years after it was bought is modelled by

$$\frac{dV}{dt} = -\frac{V}{2(t+1)}, \qquad t \geqslant 0$$

The machine was bought for $£36\,000$.

(a) Solve the differential equation to show that $V = \dfrac{36}{\sqrt{t+1}}$. [5 marks]

Answer .....

(b) Find the time at which the model predicts that the machine is worth $£9000$. [2 marks]

Answer .....

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(c) A second model for the value is $V = 36e^{-0.2t}$. After 20 years the machine is sold for $£7500$. Determine which of the two models gives the better prediction of this price. [2 marks]

Answer .....

(d) State one limitation of the model $V = \dfrac{36}{\sqrt{t+1}}$. [1 mark]

Answer .....

Total for X11: 10 marks

X12 The equation $\ln x + x^2 = 4$ is to be solved for $x > 0$.

(a) By considering the graphs of $y = \ln x$ and $y = 4 - x^2$, explain why the equation has exactly one root. [2 marks]

Answer .....

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(b) Use the iteration $x_{n+1} = \sqrt{4 - \ln x_n}$ with $x_1 = 1.8$ to find x_2, x_3 and x_4 to 4 decimal places. [3 marks]

Answer .....

(c) Let $f(x) = \ln x + x^2 - 4$. Show that the Newton-Raphson method for $f(x) = 0$ can be written as [3 marks]

$$x_{n+1} = \frac{x_n^3 + 5x_n - x_n \ln x_n}{2x_n^2 + 1}$$

Answer .....

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(d) Show that the root is 1.841 correct to 3 decimal places. [2 marks]

Answer .....

Total for X12: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a complete proof: the assumption, each step with its reason, and a final conclusion.	○	○	○
I can find my own route through a question that has no parts, and give exact answers.	○	○	○
I can set up and solve a model in context, and say what the answer means and where the model stops working.	○	○	○
I can justify a result fully: a range of validity, a nature of a turning point, a root to a stated accuracy.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Assume the sum is a rational number k and square both sides. Which surd is left?

Hint 2 Make $\sqrt{15}$ the subject. The other side is built from rational numbers only, so it is rational.

Hint 3 For (b), choose two irrational numbers whose surd parts cancel when you add. You need to say why each one is irrational and finish with a conclusion.

X2

Hint 1 Write $\operatorname{cosec} x$ as $\dfrac{1}{\sin x}$ and make each bracket a single fraction.

Hint 2 Use $1 - \sin^2 x = \cos^2 x$ and $1 - \cos^2 x = \sin^2 x$, then multiply the two fractions.

Hint 3 You should get a multiple of $\sin 2x$. What are the greatest and least values it can take?

X3

Hint 1 $(ab)^n = a^n b^n$. What power of 2 comes out of $(2 + 3x)^{-2}$?

Hint 2 For (c), you only need products that give a constant, an x term or an x^2 term.

Hint 3 For (d), the range comes from the infinite series only. Is $(1 - x)$ an infinite series?

X4

Hint 1 If A, B and C are on a straight line, $\overrightarrow{AC}$ is a multiple of $\overrightarrow{AB}$. Use the coordinate you know to find the multiple.

Hint 2 Find the three sides of triangle ACD, then an angle with the cosine rule.

Hint 3 Keep the cosine as a fraction, and use $\sin^2 A = 1 - \cos^2 A$ for an exact sine.

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X5

Hint 1 For (a), ask what letter a range should be written in, and how the domain of an inverse is related to the original function.

Hint 2 For (c), $\sqrt{x^2}$ is not always x. Try $x = -3$.

Hint 3 For (d), split into $x \geqslant 0$ and $x < 0$ and check each answer lies in its region.

X6

Hint 1 Find the coordinates of A first, using $\dfrac{dy}{dx} = 0$.

Hint 2 The region lies between the curve and the line for $0 \leqslant x \leqslant 1$. Find the area under the curve by parts.

Hint 3 The area under the line OA is a triangle. Subtract it from the area under the curve.

X7

Hint 1 A comment needs the predicted number, a comparison with 9000 and a conclusion.

Hint 2 For (c), turn the equation over to get $1 + 59e^{-0.4t}$ by itself.

Hint 3 For (e), multiply the top and the bottom of the fraction by $e^{0.4t}$, then think about the size of $e^{0.4t}$ when t is small.

X8

Hint 1 Write the 1st, 3rd and 11th terms in terms of a and d. For three consecutive terms of a geometric sequence, (middle)² = (first) × (last).

Hint 2 You may divide by d only because you are told that $d \neq 0$. Say so.

Hint 3 For (c), use the result from (a) to write the sum in terms of a only.

X9

Hint 1 Factorise y. One factor is the expression for x.

Hint 2 For (b), find two different values of t that give the same x and the same y. Each value of t gives its own gradient.

Hint 3 For (c), the tangent is parallel to the x-axis when $\dfrac{dy}{dt} = 0$ and $\dfrac{dx}{dt} \neq 0$.

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X10

Hint 1 Stationary points are where $f'(x) = 0$. When does a quadratic equation have no real roots?

Hint 2 For (b), "determine the nature" needs the second derivative at each point, with its sign.

Hint 3 For (d), write $f'(x)$ as a perfect square. What is its sign just before and just after the stationary point?

X11

Hint 1 Put V with dV and t with dt. Both sides integrate to logarithms.

Hint 2 $-\frac{1}{2}\ln(t+1)$ is $\ln\left((t+1)^{-\frac{1}{2}}\right)$. Combine it with $\ln 36$.

Hint 3 For (c), work out what each model gives at $t = 20$, in pounds, and compare both with the sale price.

X12

Hint 1 For (a), think about whether each graph goes up or down as x increases.

Hint 2 For (c), write $f'(x)$ as a single fraction first. Dividing by a fraction is multiplying by it turned over.

Hint 3 Put everything over $2x_n^2 + 1$ and collect the terms on top.

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