Hints online, answers free with an account:
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A-Level Pure A* revision 3

108 marks

Date

X Exam questions

Edexcel 9MA0 Pure style, later parts of long questions. Show all your working: an answer with no method may score no marks. The hints are at the end of the sheet. Use one at a time.

X1 Prove by contradiction that there are no positive integers x and y such that [6 marks]

$$x^2 - y^2 = 10$$

Answer .....

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X2 The points $A(-2, 1)$, $B(6, 5)$ and $C(4, -1)$ lie on a circle. [7 marks]

Find an equation of the circle. You must show all stages of your working.

Answer .....

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X3 The curve C has parametric equations [8 marks]

$$x = 3\cos t, \qquad y = 2\sin 2t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The finite region R is bounded by C and the x-axis.

Use integration to show that the area of R is 4.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X4 The sum of the first n terms of a series is given by

$$S_n = 3n^2 + 5n$$

(a) Find an expression for the nth term, u_n, and hence prove that the series is arithmetic. [4 marks]

Answer .....

(b) Find $\sum_{r=11}^{30} u_r$ [2 marks]

Answer .....

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(c) Find the value of n for which $S_n = 10u_n + 80$. [3 marks]

Answer .....

Total for X4: 9 marks

X5 In this question θ is small and measured in radians.

(a) Show that $\dfrac{1 - \cos 4\theta}{\theta \sin 3\theta} \approx \dfrac{8}{3}$. [3 marks]

Answer .....

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(b) Use small angle approximations to find an approximate solution of the equation [5 marks]

$$\cos 2\theta + 5\sin\theta = 1.48$$

You must explain why you reject any value.

Answer .....

(c) A student estimates $\sin 6^\circ$ by writing $\sin 6^\circ \approx 6$. [1 mark]
Explain the error, and write down the value the student should have used.

Answer .....

Total for X5: 9 marks

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X6 The curves $y = \sin 2x$ and $y = \cos x$ are drawn for $0 \leqslant x \leqslant \frac{\pi}{2}$. [9 marks]

The finite region R_1 is bounded by the two curves and the y-axis.
The finite region R_2 is bounded by the two curves only.

Use integration to show that R_1 and R_2 have the same area.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X7 A bowl is in the shape of a hemisphere of radius 30 cm, with its rim horizontal. Water is poured into the bowl at a constant rate of 120 cm³ per second. When the depth of the water at the centre is h cm, the volume of water, V cm³, is given by

$$V = \frac{1}{3}\pi h^2(90 - h), \qquad 0 \leqslant h \leqslant 30$$

(a) Find the rate at which the depth of the water is increasing when $h = 10$. Give your answer to 3 significant figures. [4 marks]

Answer .....

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(b) The surface of the water is a circle of radius r cm. Given that $r^2 = 60h - h^2$, find the rate at which r is increasing when $h = 10$. Give your answer to 3 significant figures. [4 marks]

Answer .....

(c) Explain, with reference to the model, why the depth of the water increases more slowly as the bowl fills. [2 marks]

Answer .....

Total for X7: 10 marks

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X8 Water seeps into a mine at a constant rate of 12 m³ per hour. A pump removes water from the mine at a rate that is proportional to the volume of water in the mine. At time t hours after the pump is switched on there are x m³ of water in the mine. Initially there is no water in the mine.

When $x = 20$, the volume of water in the mine is increasing at 7 m³ per hour.

(a) Show that $\dfrac{dx}{dt} = \dfrac{48 - x}{4}$. [2 marks]

Answer .....

(b) Solve the differential equation to find x in terms of t. [5 marks]

Answer .....

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(c) Find the time at which there are 30 m³ of water in the mine. Give your answer to 3 significant figures. [2 marks]

Answer .....

(d) Interpret, in context, the value that x approaches as t becomes large. [1 mark]

Answer .....

Total for X8: 10 marks

X9 The equation $x^3 + 4x - 7 = 0$ has exactly one real root, α.

(a) Show that the equation can be written as $x = \dfrac{7}{x^2 + 4}$. [1 mark]

Answer .....

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(b) Use the iteration $x_{n+1} = \dfrac{7}{x_n^2 + 4}$ with $x_1 = 1.2$ to find x_2, x_3 and x_4 to 4 decimal places. [3 marks]

Answer .....

(c) The equation can also be written as $x = \dfrac{7 - x^3}{4}$. With $x_1 = 1.2$, the iteration $x_{n+1} = \dfrac{7 - x_n^3}{4}$ does not converge to α. Find x_2, x_3 and x_4 for this iteration and, by considering the gradient of $y = \dfrac{7 - x^3}{4}$ near α, explain why it does not converge. [3 marks]

Answer .....

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(d) Describe how the values found in part (b) approach α. [1 mark]

Answer .....

(e) Show that $\alpha = 1.255$ correct to 3 decimal places. [2 marks]

Answer .....

Total for X9: 10 marks

X10 The temperature, $\theta\,^{\circ}\mathrm{C}$, inside an oven t minutes after it is switched on is modelled by

$$\theta = 220 - 195e^{-0.12t}, \qquad t \geqslant 0$$

(a) State the temperature inside the oven when it is switched on. [1 mark]

Answer .....

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(b) Find the time at which the temperature reaches $180\,^{\circ}\mathrm{C}$. Give your answer to 3 significant figures. [3 marks]

Answer .....

(c) Find the rate at which the temperature is increasing when $t = 5$. [2 marks]

Answer .....

(d) Show that $\dfrac{d\theta}{dt} = 0.12(220 - \theta)$. [2 marks]

Answer .....

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(e) State the limiting temperature predicted by the model, and give one reason why the model may not be accurate in practice. [2 marks]

Answer .....

Total for X10: 10 marks

X11 The function f is defined by $f(x) = x^2 - 6x + 10$, $x \in \mathbb{R}$.

(a) Explain why f does not have an inverse. [2 marks]

Answer .....

(b) The function g is defined by $g(x) = x^2 - 6x + 10$, $x \geqslant a$. Given that g^{-1} exists, state the least possible value of a. [1 mark]

Answer .....

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(c) For this value of a, find $g^{-1}(x)$ and state its domain. [4 marks]

Answer .....

(d) Solve the equation $g(x) = g^{-1}(x)$. [3 marks]

Answer .....

Total for X11: 10 marks

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X12

$$f(x) = \sqrt{4 - 5x}$$

(a) Find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2, and state the set of values of x for which it is valid. [5 marks]

Answer .....

(b) By substituting $x = \frac{1}{10}$ into your expansion, show that $\sqrt{14} \approx \frac{479}{128}$. [3 marks]

Answer .....

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(c) A student substitutes $x = -1$ into the expansion to estimate $\sqrt{9}$ and obtains 2.859, not 3.
Explain why the student's method does not give a good estimate. [2 marks]

Answer .....

Total for X12: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a complete proof: the assumption, each step with its reason, and a final conclusion.	○	○	○
I can find my own route through a question that has no parts, and give exact answers.	○	○	○
I can set up and solve a model in context, and say what the answer means and where the model stops working.	○	○	○
I can justify a result fully: a range of validity, a nature of a turning point, a root to a stated accuracy.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Start "Assume there are positive integers x and y such that ..." and factorise the left-hand side.

Hint 2 The two factors $x - y$ and $x + y$ differ by $2y$. What does that say about whether they are odd or even?

Hint 3 Either look at odd $\times$ odd and even $\times$ even, or list every way of writing 10 as a product of two positive integers and solve for x each time. Do not miss a pair.

X2

Hint 1 Before starting any long algebra, look at the triangle ABC. Find the gradients of its sides.

Hint 2 If two of the sides are perpendicular, which circle theorem tells you where the diameter is?

Hint 3 The centre is the midpoint of the diameter, and the radius is the distance from the centre to any of the three points.

X3

Hint 1 Area $= \displaystyle\int y\,dx = \int y\,\frac{dx}{dt}\,dt$. Find $\dfrac{dx}{dt}$.

Hint 2 Find the value of t at each end of the curve. As x goes from 0 to 3, which way does t go?

Hint 3 Use $\sin 2t = 2\sin t\cos t$. Then the integrand is a power of $\sin t$ multiplied by $\cos t$.

X4

Hint 1 The nth term is what is added to S_{n-1} to make S_n.

Hint 2 For (b), the sum from the 11th term to the 30th term is S_{30} with the first ten terms removed.

Hint 3 For (c), substitute your expression for u_n and solve the quadratic. n must be a positive whole number.

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X5

Hint 1 For small θ in radians, $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$. Replace θ by 4θ or 3θ where needed.

Hint 2 For (b), the approximations turn the equation into a quadratic in θ.

Hint 3 A quadratic has two roots. Is each root a small angle?

X6

Hint 1 Find where the curves meet. Factorise: dividing by $\cos x$ loses a solution.

Hint 2 On each interval, decide which curve is on top. The area is the integral of (top $-$ bottom).

Hint 3 Integrating $\sin 2x$ gives $-\dfrac{1}{2}\cos 2x$. Work out both areas exactly and compare them.

X7

Hint 1 You are given $\dfrac{dV}{dt}$ and want $\dfrac{dh}{dt}$. Which derivative connects them?

Hint 2 For (b), differentiate $r^2 = 60h - h^2$ implicitly, then use the chain rule with your answer to (a).

Hint 3 For (c), look at how $\dfrac{dV}{dh}$ changes as h goes from 0 to 30.

X8

Hint 1 Rate of change $=$ rate in $-$ rate out. The rate out is kx.

Hint 2 Integrating $\dfrac{1}{48 - x}$ gives $-\ln(48 - x)$. The minus sign matters.

Hint 3 For (d), what happens to $e^{-\frac{1}{4}t}$ as t gets large? Say what the limiting value means for the water in the mine.

X9

Hint 1 For (c), differentiate $\dfrac{7 - x^3}{4}$ and put in a value of x close to the root.

Hint 2 Compare the size of that gradient with the gradient of the line $y = x$.

Hint 3 For 3 decimal places, test the two ends of the interval $[1.2545, 1.2555]$.

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X10

Hint 1 Get $e^{-0.12t}$ by itself before taking logarithms.

Hint 2 The rate of increase is $\dfrac{d\theta}{dt}$. Differentiating $-195e^{-0.12t}$ gives a positive multiple of $e^{-0.12t}$.

Hint 3 For (d), write $e^{-0.12t}$ in terms of θ from the model, or work from the right-hand side.

X11

Hint 1 For (a), find two different values of x that give the same value of $f(x)$. Completing the square shows where to look.

Hint 2 For (c), when you take the square root there are two signs. The domain of g tells you which one to keep. The domain of g^{-1} is the range of g.

Hint 3 For (d), a function and its inverse are reflections in $y = x$. Solve $g(x) = x$, then check each root against the domain.

X12

Hint 1 Take the 4 out of the bracket first: $(4-5x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1-\dfrac{5x}{4}\right)^{\frac{1}{2}}$.

Hint 2 For (b), work out the exact value of $f\left(\dfrac{1}{10}\right)$ and write it using $\sqrt{14}$.

Hint 3 For (c), compare $x = -1$ with the range of validity you stated in (a).

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