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A-Level Pure A* revision 4

102 marks

Date

X Exam questions

Edexcel 9MA0 Pure style, later parts of long questions. Show all your working: an answer with no method may score no marks. The hints are at the end of the sheet. Use one at a time.

X1 Prove that $n^4 - n^2$ is divisible by 12 for every integer n. [6 marks]

Answer .....

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X2 The first three terms of a geometric series are [7 marks]

$$(2k - 6), \qquad k, \qquad (k - 4)$$

where k is a constant.

Given that the series is convergent, find its sum to infinity. You must give a reason for rejecting any value of k.

Answer .....

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X3 Solve, for $0 < \theta < \pi$, the equation [8 marks]

$$\operatorname{cosec} 2\theta + \cot 2\theta = 3\tan\theta - 2$$

Give exact answers where possible, and other answers to 3 significant figures.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X4 Find the exact value of [8 marks]

$$\int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx$$

Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X5

$$f(x) = x^3 - 2x^2 - 0.76x + 2.24$$

(a) A student finds $f(1)$ and $f(2)$ and concludes that $f(x) = 0$ has no roots in the interval $[1, 2]$. By also considering $f(1.5)$, show that the student is wrong, and state the least number of roots in $[1, 2]$. [4 marks]

Answer .....

(b) Taking $x_1 = 1.5$, apply the Newton-Raphson method once to find x_2. [2 marks]

Answer .....

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(c) Explain, with reference to the curve $y = f(x)$, why x_2 is not close to either of the roots in $[1, 2]$. [2 marks]

Answer .....

Total for X5: 8 marks

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X6 Relative to a fixed origin O, the points A and B have position vectors $4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$ and $10\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}$ respectively. [8 marks]
The point P lies on AB such that $AP : PB = 2 : 1$.
The point Q lies on OB such that PQ is parallel to OA.

Find the exact length of PQ.

Answer .....

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X7 The functions f and g are defined by

$$f(x) = 3 + 2e^{-x}, \quad x \geqslant 0 \qquad\qquad g(x) = \frac{1}{x-3}, \quad x > 3$$

(a) Find the range of f. [2 marks]

Answer .....

(b) Find $f^{-1}(x)$ and state its domain. [4 marks]

Answer .....

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(c) Find $gf(x)$ in its simplest form and state the range of gf. [2 marks]

Answer .....

Total for X7: 8 marks

X8

$$f(x) = x^2 e^{-\frac{x}{2}}, \qquad x \in \mathbb{R}$$

(a) Find the exact coordinates of the stationary points of the curve $y = f(x)$ and determine the nature of each. You must justify your answers. [5 marks]

Answer .....

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(b) Find the set of values of x for which the curve is concave. [3 marks]

Answer .....

(c) Hence state the x-coordinates of the points of inflection of the curve, giving a reason. [1 mark]

Answer .....

Total for X8: 9 marks

X9 The curve C has parametric equations

$$x = t + \frac{4}{t}, \qquad y = t - \frac{4}{t}, \qquad t > 0$$

(a) Show that a Cartesian equation of C is $x^2 - y^2 = 16$. [2 marks]

Answer .....

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(b) Find the range of values of x on C, showing your working. [3 marks]

Answer .....

(c) Find an equation of the tangent to C at the point P where $t = 4$. Give your answer in the form $ax + by = c$, where a, b and c are integers. [3 marks]

Answer .....

(d) Show that this tangent does not meet C at any other point. [2 marks]

Answer .....

Total for X9: 10 marks

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X10 The number of insects in a colony, P, at time t months after the start of a study is modelled by

$$\frac{dP}{dt} = \frac{\pi}{12} P \sin\left(\frac{\pi t}{6}\right)$$

When $t = 0$, $P = 500$.

(a) Solve the differential equation to show that $P = 500e^{\frac{1}{2}\left(1-\cos\frac{\pi t}{6}\right)}$. [5 marks]

Answer .....

(b) Find the greatest number of insects predicted by the model, to the nearest whole number, and the first time at which it occurs. [2 marks]

Answer .....

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(c) Find the rate at which the number of insects is increasing when $t = 3$. Give your answer to the nearest whole number. [2 marks]

Answer .....

(d) Give one reason why this model may not be suitable for predicting the number of insects over several years. [1 mark]

Answer .....

Total for X10: 10 marks

X11 The values, in pounds, of two cars t years after they were bought are modelled by

$$A = 18\,000e^{-0.2t} \qquad \text{and} \qquad B = 14\,000e^{-0.12t}$$

(a) Find the rate at which the value of car A is decreasing when $t = 2$. Give your answer to the nearest pound per year. [2 marks]

Answer .....

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(b) Find the time at which the two cars have the same value. Give your answer to 3 significant figures. [3 marks]

Answer .....

(c) Find the time taken for the value of car A to fall to half its initial value. [2 marks]

Answer .....

(d) The model for car B is refined to $B = 1500 + 12\,500e^{-0.12t}$. Show that the refined model gives the same initial value, and state the value that it predicts in the long term. [2 marks]

Answer .....

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(e) Explain why the refined model is more realistic than the original model for large values of t. [1 mark]

Answer .....

Total for X11: 10 marks

X12

$$h(x) = \frac{2x^2 + x + 9}{(1 + x)(3 - 2x)}$$

(a) Find the values of the constants A, B and C such that [4 marks]

$$h(x) = A + \frac{B}{1 + x} + \frac{C}{3 - 2x}$$

Answer .....

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(b) Hence find the binomial expansion of $h(x)$, in ascending powers of x, up to and including the term in x^2. [4 marks]

Answer .....

(c) Find, in terms of n, the coefficient of x^n in the expansion of $h(x)$, where $n \geqslant 1$. [2 marks]

Answer .....

Total for X12: 10 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a complete proof: the assumption, each step with its reason, and a final conclusion.	○	○	○
I can find my own route through a question that has no parts, and give exact answers.	○	○	○
I can set up and solve a model in context, and say what the answer means and where the model stops working.	○	○	○
I can justify a result fully: a range of validity, a nature of a turning point, a root to a stated accuracy.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Factorise fully first. $12 = 3 \times 4$, so aim to show the expression is a multiple of 3 and also a multiple of 4.
	Hint 2	$n - 1$, n and $n + 1$ are consecutive. What must be true of one of any three consecutive integers?
	Hint 3	For the factor of 4, take n even and n odd as two cases. Finish with one sentence that covers all integers.
X2	**Hint 1**	In a geometric series, (second term) $\div$ (first term) $=$ (third term) $\div$ (second term).
	Hint 2	You should get two values of k. Work out the common ratio for each one.
	Hint 3	A geometric series converges only when $\lvert r \rvert < 1$. Which value of k gives this?
X3	**Hint 1**	Write the left-hand side as one fraction over $\sin 2\theta$.
	Hint 2	Choose the form of $\cos 2\theta$ that removes the 1 from the numerator. The left-hand side becomes a single function of θ.
	Hint 3	Write everything in $\tan\theta$ and solve a quadratic. Both roots give an angle between 0 and π.
X4	**Hint 1**	Use integration by parts. Call the integral I.
	Hint 2	Use parts a second time, choosing u and $\dfrac{dv}{dx}$ the same way round as the first time.
	Hint 3	The original integral I appears again on the right. Collect the I terms on one side and solve for I.

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X5	**Hint 1**	The same sign at both ends of an interval can hide an even number of roots.
	Hint 2	Split $[1, 2]$ into two intervals at 1.5 and look for a change of sign in each.
	Hint 3	For (c), think about what the tangent at $x = 1.5$ looks like when the gradient is -0.01.
X6	**Hint 1**	Find the position vector of P first: go from O to A, then two thirds of the way along $\overrightarrow{AB}$.
	Hint 2	Q is on OB, so its position vector is $\lambda\overrightarrow{OB}$. Write $\overrightarrow{PQ}$ in terms of λ.
	Hint 3	"Parallel to OA" means $\overrightarrow{PQ}$ is a multiple of $\overrightarrow{OA}$. Compare two components to find λ, and check the third.
X7	**Hint 1**	For (a), find the value at $x = 0$ and what happens as x gets large. One end of the range is reached and the other is not.
	Hint 2	For (b), get e^{-x} on its own before taking logarithms. The domain of f^{-1} is the range you found in (a).
	Hint 3	For (c), put $f(x)$ into g and simplify. The domain of gf is the domain of f, which is $x \geqslant 0$.
X8	**Hint 1**	Use the product rule and take out the factor $e^{-\frac{x}{2}}$, which is never zero.
	Hint 2	"Justify" means a second derivative with its sign at each point, or the sign of $f'(x)$ on each side.
	Hint 3	A curve is concave where $f''(x) < 0$. $f''(x) = 0$ alone is not enough for a point of inflection: the sign must change.
X9	**Hint 1**	Add the two equations and subtract them. What do you get when you multiply the results?
	Hint 2	For (b), x is a function of t. Find its minimum value for $t > 0$ and justify that it is a minimum.
	Hint 3	For (d), solve the tangent and the Cartesian equation together. What kind of root do you get?

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X10

Hint 1 Divide by P and integrate both sides. $\int \sin\left(\frac{\pi t}{6}\right) dt = -\frac{6}{\pi}\cos\left(\frac{\pi t}{6}\right)$.

Hint 2 For (b), the power of e is largest when the cosine is as small as it can be.

Hint 3 For (c), you do not need to differentiate. The differential equation gives the rate once you know P and t.

X11

Hint 1 "Rate of decrease" means differentiate. State the answer as a positive rate of decrease, with units.

Hint 2 For (b), divide both sides by $e^{-0.2t}$ so there is one exponential.

Hint 3 For (e), compare what each model says about a very old car.

X12

Hint 1 The top and the bottom are both quadratics, so there is a constant term A as well as the two fractions. Compare coefficients of x^2 to find it.

Hint 2 For (b), $6(3-2x)^{-1} = 6 \times 3^{-1}\left(1-\frac{2x}{3}\right)^{-1}$.

Hint 3 For (c), $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots$ and $(1-y)^{-1} = 1 + y + y^2 + \ldots$. Write down the term in x^n of each.

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