EDEXCEL A-LEVEL MATHEMATICS (9MA0) · PURE PAPERS 1 AND 2 · A-LEVEL PURE A* REVISION · SHEET 6 OF 10

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-a-star-revision-6/

A-Level Pure A* revision 6

108 marks

Date

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X Exam questions

Edexcel 9MA0 Pure style, later parts of long questions. Show all your working: an answer with no method may score no marks. The hints are at the end of the sheet. Use one at a time.

X1 Prove by contradiction that the equation [6 marks]

$$x^3 + x + 1 = 0$$

has no rational solutions.

Answer .....

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X2 [7 marks]

$$h(\theta) = \frac{39}{8 + 3\cos 2\theta - 4\sin 2\theta}, \quad \theta \geqslant 0$$

where θ is in radians.

Find the maximum value of $h(\theta)$ and the smallest value of θ at which this maximum occurs. Give θ to 3 significant figures.

Answer .....

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X3 The points A, B and C have coordinates $(2, -1, 4)$, $(5, 3, 4)$ and $(9, 0, 16)$ respectively. [8 marks]

Prove that angle ABC is a right angle. Given that $ABCD$ is a rectangle, find the coordinates of D and the area of the rectangle.

Answer .....

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X4 A geometric series has first term 5 and common ratio $\dfrac{2x-3}{4}$.

(a) Find the set of values of x for which the series is convergent. [3 marks]

Answer .....

(b) Given that the sum to infinity of the series is 8, find the value of x. [3 marks]

Answer .....

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(c) Show that, for every value of x for which the series is convergent, the sum to infinity is greater than $\frac{5}{2}$. [2 marks]

Answer .....

Total for X4: 8 marks

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X5 Show that [9 marks]

$$\int_0^2 \frac{4x^2 + 5x + 3}{(x+1)^2(2x+1)}\,dx = \ln p - q$$

where p and q are rational numbers to be found.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X6

$$f(x) = x \ln x - 2, \qquad x > 0$$

(a) Show that $f(x) = 0$ has a root α in the interval $[2.3, 2.4]$. [2 marks]

Answer .....

(b) Taking $x_1 = 2.3$, apply the Newton-Raphson method once to find a second approximation to α. Give your answer to 4 decimal places. [3 marks]

Answer .....

(c) Show that $\alpha = 2.346$ correct to 3 decimal places. [2 marks]

Answer .....

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(d) With $x_1 = 0.2$, find x_2 to 2 decimal places, and explain why the Newton-Raphson method cannot be continued. [2 marks]

Answer .....

Total for X6: 9 marks

X7 A metal bar is taken from a furnace and left to cool in a room at a constant temperature of $A\,^\circ\mathrm{C}$. The temperature of the bar, $\theta\,^\circ\mathrm{C}$, at time t minutes satisfies

$$\frac{d\theta}{dt} = -k(\theta - A), \qquad \theta > A$$

where k is a positive constant.

(a) Show that $\theta = A + Be^{-kt}$, where B is a constant. [4 marks]

Answer .....

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(b) The temperature of the bar is $86\,^{\circ}\mathrm{C}$ at $t = 0$, $54\,^{\circ}\mathrm{C}$ at $t = 10$ and $38\,^{\circ}\mathrm{C}$ at $t = 20$. Find the value of A. [4 marks]

Answer .....

(c) Find the exact value of k, and the time at which the temperature of the bar is $30\,^{\circ}\mathrm{C}$. [2 marks]

Answer .....

Total for X7: 10 marks

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X8 The number of deer, P, in a forest t years after a study began is modelled by

$$P = \frac{600}{1 + 5e^{-0.4t}}, \qquad t \geqslant 0$$

(a) State the number of deer at the start of the study. [1 mark]

Answer .....

(b) Find the time at which the model predicts 400 deer. Give your answer to 3 significant figures. [3 marks]

Answer .....

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(c) Show that $\frac{dP}{dt} = \frac{1200e^{-0.4t}}{(1+5e^{-0.4t})^2}$, and hence explain why the number of deer is always increasing. [3 marks]

Answer .....

(d) State the limiting number of deer predicted by the model. [1 mark]

Answer .....

(e) Given that the number of deer is increasing most quickly when $P = 300$, find the time at which this happens. [2 marks]

Answer .....

Total for X8: 10 marks

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X9 The function g is defined by $g(x) = 7 - 2|x - 1|$, $x \in \mathbb{R}$.

(a) State the range of g. [1 mark]

Answer .....

(b) Find $gg(4)$. [2 marks]

Answer .....

(c) The equation $g(x) = k$, where k is a constant, has two distinct roots α and β.
Given that $\beta - \alpha = 5$, find the value of k. [3 marks]

Answer .....

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(d) Using algebra, solve the inequality $g(x) > |x|$. [4 marks]

Answer .....

Total for X9: 10 marks

X10

$$f(x) = \frac{4 + x}{\sqrt{1 - 2x}}, \quad |x| < \frac{1}{2}$$

(a) Show that, when x is small, $f(x) \approx 4 + 5x + 7x^2$. [4 marks]

Answer .....

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(b) By substituting $x = \dfrac{1}{18}$, show that $\sqrt{2} \approx \dfrac{2786}{1971}$. [4 marks]

Answer .....

(c) Without calculating $\sqrt{2}$, explain whether $\dfrac{2786}{1971}$ is an over-estimate or an under-estimate of $\sqrt{2}$. [2 marks]

Answer .....

Total for X10: 10 marks

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X11 The curve C has parametric equations

$$x = (t+1)^2, \qquad y = \frac{6}{t+3}, \qquad 0 \leqslant t \leqslant 3$$

The finite region R is bounded by C, the x-axis and the lines $x = 1$ and $x = 16$.

(a) Show that the area of R is given by $12\displaystyle\int_0^3 \frac{t+1}{t+3}\,dt$. [3 marks]

Answer .....

(b) Hence find the exact area of R, giving your answer in the form $a + b\ln 2$, where a and b are integers. [4 marks]

Answer .....

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(c) Find a Cartesian equation of C in the form $y = f(x)$, and state the domain of f. [3 marks]

Answer .....

Total for X11: 10 marks

X12 The curve C has equation

$$y^2 + 4x \ln y = 8x - 7, \qquad y > 0$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{2y(2 - \ln y)}{y^2 + 2x}$. [4 marks]

Answer .....

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(b) The normal to C at the point $P(1, 1)$ meets the x-axis at A and the y-axis at B. Find the exact area of triangle OAB, where O is the origin. [4 marks]

Answer .....

(c) Show that there is no point on C at which the tangent is parallel to the x-axis. [3 marks]

Answer .....

Total for X12: 11 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a complete proof: the assumption, each step with its reason, and a final conclusion.	○	○	○
I can find my own route through a question that has no parts, and give exact answers.	○	○	○
I can set up and solve a model in context, and say what the answer means and where the model stops working.	○	○	○
I can justify a result fully: a range of validity, a nature of a turning point, a root to a stated accuracy.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 A rational number is $\frac{p}{q}$ with p and q integers. Assume the fraction is in its lowest terms, substitute it into the equation and clear the fractions.

Hint 2 You should reach $p^3 + pq^2 + q^3 = 0$. Think about whether each term is odd or even.

Hint 3 p and q cannot both be even (why?). Work through the other three cases and show the left-hand side is odd every time.

X2

Hint 1 Write $3\cos 2\theta - 4\sin 2\theta$ in the form $R\cos(2\theta + \alpha)$.

Hint 2 A fraction with a fixed positive numerator is largest when its denominator is smallest. What is the smallest value of the denominator?

Hint 3 The denominator is smallest when $\cos(2\theta + \alpha) = -1$. Solve for the smallest $\theta \geqslant 0$.

X3

Hint 1 Find the squares of the three sides of triangle ABC.

Hint 2 A proof needs a statement of why the lengths show a right angle, and at which vertex.

Hint 3 In the rectangle $ABCD$, the side AD is equal and parallel to BC.

X4

Hint 1 A geometric series converges when $|r| < 1$. Write this as a double inequality.

Hint 2 For (b), simplify $1 - \frac{2x - 3}{4}$ to a single fraction before solving.

Hint 3 For (c), work with r and not x: what values can $1 - r$ take when $-1 < r < 1$?

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X5

Hint 1 The factor $(x+1)^2$ is repeated, so you need three partial fractions.

Hint 2 $\displaystyle\int \frac{6}{2x+1}\,dx$ is $3\ln(2x+1)$, not $6\ln(2x+1)$. The $\dfrac{B}{(x+1)^2}$ term does not give a logarithm.

Hint 3 At the lower limit the bracket is not zero: $\dfrac{2}{x+1}$ is 2 when $x=0$.

X6

Hint 1 Differentiate $x\ln x$ with the product rule.

Hint 2 For 3 decimal places, test 2.3455 and 2.3465.

Hint 3 For (d), look at the sign of x_2 and at the domain of f.

X7

Hint 1 Divide by $(\theta - A)$ and integrate. When you remove the logarithm, e^{-kt+c} is $e^{c}\times e^{-kt}$.

Hint 2 For (b), write down the three equations. The readings are 10 minutes apart, so e^{-20k} is the square of e^{-10k}.

Hint 3 $\theta - A$ at the three times forms a geometric sequence. Use that to make an equation in A only.

X8

Hint 1 For (b), turn the equation over: $1+5e^{-0.4t}=\dfrac{600}{P}$.

Hint 2 Write P as $600(\ldots)^{-1}$ and use the chain rule. The derivative of $5e^{-0.4t}$ is $-2e^{-0.4t}$.

Hint 3 To explain "always increasing", say why the top and the bottom of the fraction are each positive.

X9

Hint 1 The graph is an upside-down V with its vertex at $(1,7)$.

Hint 2 For (c), solve $g(x)=k$ for $|x-1|$ first. The two roots are the same distance either side of $x=1$.

Hint 3 For (d), there are two modulus signs, so there are three regions: $x<0$, $0\leqslant x<1$ and $x\geqslant 1$. Check that each answer lies in the region it came from.

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X10

Hint 1 Write $f(x) = (4 + x)(1 - 2x)^{-\frac{1}{2}}$ and expand the second bracket to the x^2 term first.

Hint 2 For (b), $1 - 2 \times \frac{1}{18} = \frac{8}{9}$. Its square root contains $\sqrt{2}$.

Hint 3 For (c), look at the signs of the terms that were left out when x is positive.

X11

Hint 1 Area $= \int y \, \frac{dx}{dt} \, dt$. Find the values of t where $x = 1$ and $x = 16$.

Hint 2 For (b), the top and the bottom have the same degree. Write $t + 1$ as $(t + 3) - 2$.

Hint 3 For (c), make t the subject of the equation for x. Why is only one square root possible?

X12

Hint 1 $4x \ln y$ is a product. Differentiating $\ln y$ with respect to x gives $\frac{1}{y}\frac{dy}{dx}$.

Hint 2 For (b), find where the normal crosses each axis. The triangle has a right angle at O.

Hint 3 For (c), find the only value of y that makes $\frac{dy}{dx} = 0$, then test whether a point with that y lies on C.

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