EDEXCEL A-LEVEL MATHEMATICS (9MA0) · PURE PAPERS 1 AND 2 · A-LEVEL PURE A* REVISION · SHEET 7 OF 10

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-a-star-revision-7/

A-Level Pure A* revision 7

104 marks

Date

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X Exam questions

Edexcel 9MA0 Pure style, later parts of long questions. Show all your working: an answer with no method may score no marks. The hints are at the end of the sheet. Use one at a time.

X1 Use the substitution $u = 1 + x^2$ to find the exact value of [7 marks]

$$\int_0^{\sqrt{3}} \frac{x^3}{\sqrt{1+x^2}}\, dx$$

Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X2 In this question p and q are integers.

(a) Prove by contradiction that $\log_2 3$ is irrational. [4 marks]

Answer .....

(b) Explain why the same method does not prove that $\log_4 8$ is irrational, and state the exact value of $\log_4 8$. [3 marks]

Answer .....

Total for X2: 7 marks

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X3 A water tank holds 500 litres at the start of day 1. [8 marks]

During each day the tank loses 8% of the water that it held at the start of that day. At the end of each day, 30 litres of water are added.

Find the first day at the end of which the tank holds less than 400 litres.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X4 The curve C has parametric equations [8 marks]

$$x = 4\cos t, \qquad y = 2\sin 2t, \qquad 0 \leqslant t < 2\pi$$

Find the exact coordinates of each point on C at which the gradient of the tangent, with the value of t at that point, is 1. Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X5 A sample contains two radioactive isotopes. The total mass, m grams, of the two isotopes t days after the sample was prepared is modelled by

$$m = 10 \times 2^{-\frac{t}{8}} + 30 \times 2^{-\frac{t}{2}}$$

The first term is the mass of isotope X and the second term is the mass of isotope Y.

(a) State the total mass when the sample was prepared. [1 mark]

Answer .....

(b) Find the time at which the masses of the two isotopes are equal. Give your answer to 3 significant figures. [3 marks]
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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(c) Find the rate at which the total mass is decreasing when $t = 0$. Give your answer to 3 significant figures. [3 marks]

Answer .....

(d) Explain why, for large values of t, the total mass is approximately $10 \times 2^{-\frac{t}{8}}$. [2 marks]

Answer .....

Total for X5: 9 marks

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X6 The circle C has equation $(x+1)^2+(y-2)^2=25$. [9 marks]

The line with equation $x+7y-38=0$ meets C at the points A and B.

The tangents to C at A and at B meet at the point T.

Find the coordinates of T.

Answer .....

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X7 The functions f and g are defined by

$$f(x) = \frac{2x+3}{x-2}, \quad x \in \mathbb{R},\ x \neq 2 \qquad\qquad g(x) = x^2 - 3, \quad x \in \mathbb{R}$$

(a) Show that $ff(x) = x$ for all $x \neq 2$. [3 marks]

Answer .....

(b) Hence write down $f^{-1}(x)$. [1 mark]

Answer .....

(c) Find the exact values of x for which $fg(x)$ is not defined, giving a reason. [2 marks]

Answer .....

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(d) Solve the equation $fg(x) = 5$, giving your answers in the form $\pm\frac{\sqrt{a}}{b}$ where a and b are integers. [3 marks]

Answer .....

Total for X7: 9 marks

X8

$$g(x) = \frac{4 + kx}{(1 - x)(2 + x)}$$

where k is a constant.

(a) Find the binomial expansion of $\frac{1}{(1 - x)(2 + x)}$, in ascending powers of x, up to and including the term in x^3. [4 marks]

Answer .....

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(b) Given that the coefficient of x^2 in the expansion of $g(x)$ is zero, find the value of k. [3 marks]

Answer .....

(c) For this value of k, find the coefficient of x^3 in the expansion of $g(x)$. [2 marks]

Answer .....

Total for X8: 9 marks

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X9 In this question you must show all stages of your working.

(a) Prove that [3 marks]

$$\cot x - \tan x \equiv 2\cot 2x, \quad x \neq \frac{n\pi}{2}$$

Answer .....

(b) Hence solve, for $0 < x < \pi$, the equation $\cot x - \tan x = 2\sqrt{3}$. Give your answers in terms of π. [3 marks]

Answer .....

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(c) Hence show that $\tan \frac{\pi}{12} = 2 - \sqrt{3}$. [3 marks]

Answer .....

Total for X9: 9 marks

X10 In this question you must show all stages of your working.

(a) Using the small angle approximations for $\sin\theta$ and $\cos\theta$, prove from first principles that the derivative of $\cos 2x$ is $-2\sin 2x$. [5 marks]

Answer .....

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(b) The curve C has equation $y = x + \cos 2x$, $0 \leqslant x \leqslant \pi$. Find the exact x-coordinates of the stationary points of C and determine the nature of each. [4 marks]

Answer .....

Total for X10: 9 marks

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X11 In a chemical reaction the mass, m grams, of a substance remaining at time t minutes is modelled by

$$\frac{dm}{dt} = -km^2$$

where k is a positive constant. When $t = 0$, $m = 50$, and when $t = 4$, $m = 25$.

(a) Solve the differential equation to show that $m = \dfrac{200}{t + 4}$. [6 marks]

Answer .....

(b) Find the time at which 10 grams of the substance remain. [1 mark]

Answer .....

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(c) A student says, "The mass halves every 4 minutes." Show that the student is wrong. [2 marks]

Answer .....

(d) Describe what the model predicts about the mass of the substance in the long term. [1 mark]

Answer .....

Total for X11: 10 marks

X12 The equation $e^x = 3x + 2$ has two roots: a positive root α and a negative root β.
Let $f(x) = e^x - 3x - 2$.

(a) Show that α lies in the interval $[2, 2.2]$. [2 marks]

Answer .....

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(b) Use the iteration $x_{n+1} = \ln(3x_n + 2)$ with $x_1 = 2$ to find x_2, x_3 and x_4 to 4 decimal places. [3 marks]

Answer .....

(c) Describe the way in which these values approach α. [1 mark]

Answer .....

(d) Explain why $x_1 = -0.7$ cannot be used as a starting value for the iteration in part (b). [1 mark]

Answer .....

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(e) The iteration $x_{n+1} = \dfrac{e^{x_n} - 2}{3}$ with $x_1 = -0.4$ converges to β. [3 marks]

Find x_2, x_3 and x_4 to 4 decimal places and, by choosing a suitable interval, show that $\beta = -0.455$ correct to 3 decimal places.

Answer .....

Total for X12: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a complete proof: the assumption, each step with its reason, and a final conclusion.	○	○	○
I can find my own route through a question that has no parts, and give exact answers.	○	○	○
I can set up and solve a model in context, and say what the answer means and where the model stops working.	○	○	○
I can justify a result fully: a range of validity, a nature of a turning point, a root to a stated accuracy.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	$du = 2x\,dx$ uses one factor of x. What is left of x^3?
	Hint 2	Replace the remaining x^2 using $x^2 = u - 1$, so the integral contains only u.
	Hint 3	Change both limits to values of u, then write the integrand as two powers of u.
X2	**Hint 1**	Assume $\log_2 3 = \dfrac{p}{q}$ and rewrite it without a logarithm. Raise both sides to the power q.
	Hint 2	Compare 2^p and 3^q: one is even and one is odd. Say why p cannot be 0.
	Hint 3	For (b), follow the same steps with 4 and 8. Both are powers of 2. Does a contradiction appear?
X3	**Hint 1**	Write down the amount at the end of day 1, day 2 and day 3 without simplifying the powers of 0.92. Look for the pattern.
	Hint 2	The 30 litres added each day give a geometric series $30 + 30(0.92) + 30(0.92)^2 + \ldots$
	Hint 3	You should reach an inequality of the form $0.92^n < \ldots$. Take care with the inequality sign when you divide by $\ln 0.92$.
X4	**Hint 1**	Find $\dfrac{dy}{dx}$ in terms of t and set it equal to 1.
	Hint 2	Use a double angle formula to make the equation a quadratic in $\sin t$.
	Hint 3	There are three values of t in the range. Substitute each into both x and y, giving exact values.

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X5

Hint 1 For (b), divide both sides by $10 \times 2^{-\frac{t}{2}}$ so that all the powers of 2 are on one side.

Hint 2 $\dfrac{d}{dt}\left(2^{kt}\right) = k \ln 2 \times 2^{kt}$.

Hint 3 For (d), work out both terms at a large value of t and compare them.

X6

Hint 1 Find A and B first. Writing the line as $x = 38 - 7y$ avoids fractions.

Hint 2 A tangent is perpendicular to the radius at the point of contact.

Hint 3 Write each tangent in the form $ax + by = c$ and solve the two equations together.

X7

Hint 1 For (a), replace every x in $f(x)$ by $\dfrac{2x+3}{x-2}$, then multiply the top and the bottom by $(x-2)$.

Hint 2 If doing f twice returns x, what does that say about f and f^{-1}?

Hint 3 For (d), rationalise at the end: $\sqrt{\dfrac{22}{3}} = \dfrac{\sqrt{22}}{\sqrt{3}}$, then multiply the top and the bottom by $\sqrt{3}$.

X8

Hint 1 Split $\dfrac{1}{(1-x)(2+x)}$ into partial fractions, then expand each one. Remember the factor 2^{-1}.

Hint 2 For (b), the x^2 term of a product comes from (constant) $\times$ (x^2 term) and (x term) $\times$ (x term).

Hint 3 For (c), use the same idea for x^3: which pairs of terms multiply to give x^3?

X9

Hint 1 Write $\cot x$ and $\tan x$ in terms of $\sin x$ and $\cos x$ and put them over a common denominator.

Hint 2 For (b), the range for $2x$ is $0 < 2x < 2\pi$. There are two values of $2x$.

Hint 3 For (c), one of your solutions to (b) is $\dfrac{\pi}{12}$. Put $t = \tan\dfrac{\pi}{12}$ into the equation and solve for t.

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X10

Hint 1 Expand $\cos(2x + 2h)$ with the addition formula. The approximations are applied to $\sin 2h$ and $\cos 2h$, so the angle is $2h$.

Hint 2 $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$ with $\theta = 2h$ gives $1 - 2h^2$. Divide by h before letting $h \to 0$.

Hint 3 For (b), there are two values of $2x$ between 0 and 2π with $\sin 2x = \dfrac{1}{2}$. Use the second derivative at each.

X11

Hint 1 Integrate m^{-2} with respect to m. The result is $-\dfrac{1}{m}$.

Hint 2 There are two unknowns, c and k, and two conditions. Use one for each.

Hint 3 For (c), find when the mass is 12.5 g and compare the two time intervals.

X12

Hint 1 Compare the values in (b) with each other and with the interval in (a). Are they all on one side of the root?

Hint 2 For (d), work out $3x_1 + 2$ before taking the logarithm.

Hint 3 For 3 decimal places, the interval is $[-0.4555, -0.4545]$. Use f, not the iteration formula.

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