EDEXCEL A-LEVEL MATHEMATICS (9MA0) · PURE PAPERS 1 AND 2 · A-LEVEL PURE A* REVISION · SHEET 8 OF 10

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-a-star-revision-8/

A-Level Pure A* revision 8

103 marks

Date

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X Exam questions

Edexcel 9MA0 Pure style, later parts of long questions. Show all your working: an answer with no method may score no marks. The hints are at the end of the sheet. Use one at a time.

X1 In this question n, x and y are integers.

(a) Prove that n^2 is either a multiple of 3 or one more than a multiple of 3. [4 marks]

Answer .....

(b) Hence prove that there are no integers x and y such that $x^2 = 3y^2 + 2$. [3 marks]

Answer .....

Total for X1: 7 marks

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X2 Use calculus to prove that [7 marks]

$$\ln x \leqslant \frac{x^2}{2e} \qquad \text{for all } x > 0$$

Answer .....

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X3 A circular patch of oil on water has radius r cm at time t minutes. [8 marks]
The rate of increase of the radius is modelled as being inversely proportional to the square of the radius.
Initially the radius is 1 cm, and after 9 minutes the radius is 4 cm.

Find the time at which the model predicts that the area of the patch is 64π cm².

Answer .....

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X4

$$f(x) = x^2 - 4\cos x$$

where x is in radians. The equation $f(x) = 0$ has a root α in the interval $[1.2, 1.3]$.

(a) Show that $f(x) = 0$ has a root in $[1.2, 1.3]$. [2 marks]

Answer .....

(b) Taking $x_1 = 1.2$, apply the Newton-Raphson method once to find a second approximation to α. Give your answer to 4 decimal places. [3 marks]

Answer .....

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(c) A student writes: "$f(1.19) < 0$ and $f(1.21) > 0$, so $\alpha = 1.20$ correct to 2 decimal places." Explain why the student's working does not justify this conclusion, and give a correct justification. [3 marks]

Answer .....

Total for X4: 8 marks

X5

$$f(x) = \sqrt[3]{8 + 3x}$$

(a) Find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2, and state the set of values of x for which the expansion is valid. [5 marks]

Answer .....

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(b) Use your expansion with a suitable value of x to find an approximation for $\sqrt[3]{9}$. Give your answer as a fraction. [2 marks]

Answer .....

(c) Explain why the expansion cannot be used with $x = 3$ to estimate $\sqrt[3]{17}$. [1 mark]

Answer .....

Total for X5: 8 marks

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X6 Solve, for $-\pi < x \leqslant \pi$, the equation [8 marks]

$$3\sin 2x = 4\tan x$$

Give exact answers where possible, and other answers to 3 significant figures.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X7 The points A, B and C have coordinates $(2, 3, -1)$, $(6, 1, 3)$ and $(k, 2k - 1, 3)$ respectively, where k is a positive constant. [9 marks]

Given that $AB = AC$, find the exact area of triangle ABC.

Answer .....

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X8 The function h is defined by

$$h(x) = \frac{12}{x^2 - 4x + 7}, \quad x \in \mathbb{R}$$

(a) Find the range of h. [3 marks]

Answer .....

(b) The function k is defined by $k(x) = \frac{12}{x^2 - 4x + 7}$, $x \geqslant 2$. [4 marks]

Find $k^{-1}(x)$ and state its domain.

Answer .....

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(c) Find the value of x for which $k^{-1}(x) = 5$. [2 marks]

Answer .....

Total for X8: 9 marks

X9 A geometric series has first term a and common ratio r, where $r \neq 1$.

(a) Prove that the sum of the first n terms is $S_n = \dfrac{a(1 - r^n)}{1 - r}$. [3 marks]

Answer .....

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(b) For a particular geometric series, the sum to infinity is 125 and the sum of the first three terms is 117.
Find the value of r and the value of a. [3 marks]

Answer .....

(c) Find the least value of n for which $S_\infty - S_n < 0.001$. [3 marks]

Answer .....

Total for X9: 9 marks

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X10 The population, P, of a town t years after the start of 2015 is modelled by $P = 24\,000 \times 1.03^t$.

(a) State the percentage by which the population increases each year. [1 mark]

Answer .....

(b) Find the year during which the model predicts that the population first exceeds $40\,000$. [3 marks]

Answer .....

(c) The model can be written as $P = 24\,000e^{kt}$. Find the value of k to 3 significant figures. [1 mark]

Answer .....

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(d) Find the rate at which the population is increasing when $t = 10$. Give your answer to the nearest whole number. [2 marks]

Answer .....

(e) The population of a second town is modelled by $Q = 30\,000 \times 1.012^t$. Find the year during which the population of the first town overtakes the population of the second town. [3 marks]

Answer .....

Total for X10: 10 marks

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X11 A jet of water leaves a fountain nozzle. Relative to a fixed origin O on horizontal ground, the position (x, y) of the water, in metres, t seconds after it leaves the nozzle is modelled by

$$x = 6t, \qquad y = 1 + 8t - 5t^2$$

where x is the horizontal distance and y is the height above the ground.

(a) Find a Cartesian equation for the path of the water. [2 marks]

Answer .....

(b) Find the horizontal distance from O at which the water hits the ground, and hence state the domain for which your equation in (a) is a valid model. [3 marks]

Answer .....

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(c) Find the greatest height of the water above the ground. [2 marks]

Answer .....

(d) Find the angle that the path of the water makes with the horizontal at the moment the water hits the ground. Give your answer to the nearest 0.1°. [3 marks]

Answer .....

Total for X11: 10 marks

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X12 The curve C has equation $y = x \sin 2x,\ 0 \leqslant x \leqslant \dfrac{\pi}{2}$.

(a) Find $\displaystyle\int x \sin 2x \, dx$. [3 marks]

Answer .....

(b) Find the exact area of the finite region bounded by C and the x-axis. [2 marks]

Answer .....

(c) Show that the line $y = x$ is a tangent to C at the point where $x = \dfrac{\pi}{4}$. [2 marks]

Answer .....

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(d) Find the exact area of the finite region bounded by C and the line $y = x$ for $0 \leqslant x \leqslant \frac{\pi}{4}$. [3 marks]

Answer .....

Total for X12: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a complete proof: the assumption, each step with its reason, and a final conclusion.	○	○	○
I can find my own route through a question that has no parts, and give exact answers.	○	○	○
I can set up and solve a model in context, and say what the answer means and where the model stops working.	○	○	○
I can justify a result fully: a range of validity, a nature of a turning point, a root to a stated accuracy.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 For (a), every integer has remainder 0, 1 or 2 on division by 3. Square $3k$, $3k+1$ and $3k+2$.

Hint 2 Write each square in the form $3(\dots)$ or $3(\dots)+1$. Be careful with $(3k+2)^2$: the 4 is $3+1$.

Hint 3 For (b), what remainder does $3y^2+2$ leave on division by 3? Compare with what (a) allows for x^2.

X2

Hint 1 Move everything to one side: study the function $g(x) = \dfrac{x^2}{2e} - \ln x$.

Hint 2 Find the stationary point of g and show it is a minimum.

Hint 3 Work out the value of g at its minimum. What does that tell you about $g(x)$ everywhere else? Finish with a sentence.

X3

Hint 1 "Inversely proportional to the square of the radius" means $\dfrac{dr}{dt} = \dfrac{k}{r^2}$.

Hint 2 Multiply both sides by r^2 and integrate. Use the two conditions to find the two constants.

Hint 3 The question gives an area, but your equation is in r. Find the radius first.

X4

Hint 1 The derivative of $-4\cos x$ is $+4\sin x$. Keep the calculator in radians.

Hint 2 Which numbers round to 1.20 to 2 decimal places? Write them as an interval.

Hint 3 Does every number between 1.19 and 1.21 lie in that interval?

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X5

Hint 1 $(8+3x)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(1+\dfrac{3x}{8}\right)^{\frac{1}{3}}$, and $8^{\frac{1}{3}} = 2$.

Hint 2 For (b), choose x so that $8+3x = 9$, and check that it is inside the valid range.

Hint 3 For (c), compare $x = 3$ with the range of validity from (a).

X6

Hint 1 Write both sides in terms of $\sin x$ and $\cos x$.

Hint 2 Do not divide by $\sin x$. Bring everything to one side and factorise.

Hint 3 $\cos^2 x = \dfrac{2}{3}$ gives a positive and a negative value of $\cos x$, and each gives two angles in the range.

X7

Hint 1 Work with squares of lengths: set $AB^2 = AC^2$ to get an equation in k.

Hint 2 The equation has two solutions. Read the question again to see which one is allowed.

Hint 3 The triangle is isosceles. The line from A to the midpoint of BC is perpendicular to BC.

X8

Hint 1 Complete the square in the denominator. What is the least value of the denominator, and can it be as large as you like?

Hint 2 For (b), the domain $x \geqslant 2$ decides the sign of the square root. The domain of k^{-1} is the range of k.

Hint 3 For (c), you do not need the formula for the inverse: $k^{-1}(x) = 5$ means $x = k(5)$.

X9

Hint 1 For (a), write out S_n, multiply every term by r, and subtract one line from the other.

Hint 2 For (b), divide the formula for S_3 by the formula for S_∞.

Hint 3 For (c), $S_\infty - S_n$ simplifies to a single term. $\ln 0.4$ is negative.

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X10

Hint 1 t is measured from the start of 2015. Turn your value of t into a year.

Hint 2 For (d), it is easier to differentiate the form $24\,000e^{kt}$.

Hint 3 For (e), divide both sides by 1.012^t so that t appears once, then take logarithms.

X11

Hint 1 Make t the subject of $x = 6t$ and substitute into the equation for y.

Hint 2 The water hits the ground when $y = 0$. Only one root makes sense. The model stops being valid after that.

Hint 3 For (d), the direction of the path is given by $\dfrac{dy}{dx}$ at the landing time. The angle with the horizontal comes from its size.

X12

Hint 1 Differentiate x and integrate $\sin 2x$. Remember the factor $\dfrac{1}{2}$ when integrating $\sin 2x$.

Hint 2 For (c), two things are needed: the point is on both, and the gradients are equal there.

Hint 3 For (d), the line is above the curve. Integrate (line $-$ curve) from 0 to $\dfrac{\pi}{4}$, using your answer to (a).

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