EDEXCEL A-LEVEL MATHEMATICS (9MA0) · PURE PAPERS 1 AND 2 · A-LEVEL PURE A* REVISION · SHEET 9 OF 10

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-a-star-revision-9/

A-Level Pure A* revision 9

113 marks

Date

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X Exam questions

Edexcel 9MA0 Pure style, later parts of long questions. Show all your working: an answer with no method may score no marks. The hints are at the end of the sheet. Use one at a time.

X1 Find the exact value of [7 marks]

$$\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx$$

Give your answer in the form $\frac{\pi}{a} - \frac{1}{b}\ln 2$, where a and b are integers.
Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X2 A circle C touches the y-axis at the point $(0, 4)$ and passes through the point $P(2, 8)$. [8 marks]

Find an equation of the tangent to C at P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Answer .....

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X3 The curve C has parametric equations [8 marks]

$$x = \sin^2 t, \qquad y = 2\tan t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The finite region R is bounded by C, the x-axis and the line $x = \frac{1}{2}$.

Find the exact area of R.

Solutions relying entirely on calculator technology are not acceptable.

Answer .....

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X4 The functions f and g are defined by

$$f(x) = |x - 4|, \quad x \in \mathbb{R} \qquad\qquad g(x) = 2x + 1, \quad x \in \mathbb{R}$$

(a) Find $fg(x)$ and $gf(x)$, and state the range of each. [3 marks]

Answer .....

(b) Using algebra, solve the equation $fg(x) = gf(x)$. [4 marks]

Answer .....

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(c) Explain, with reference to the graphs of $y = fg(x)$ and $y = gf(x)$, why there is no solution with $x \geqslant 4$. [2 marks]

Answer .....

Total for X4: 9 marks

X5 An arithmetic series has first term 50 and common difference -3.

(a) Find the greatest possible value of S_n, the sum of the first n terms. You must justify that your value is the greatest. [4 marks]

Answer .....

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(b) Find the least value of n for which $S_n < 0$. [3 marks]

Answer .....

(c) Find $\sum_{r=10}^{25} u_r$ [2 marks]

Answer .....

Total for X5: 9 marks

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X6 The concentration, C mg per litre, of a drug in the blood at time t hours after it is taken is modelled by

$$C = 12te^{-\frac{1}{2}t}, \qquad t \geqslant 0$$

(a) Find the exact value of the maximum concentration and the time at which it occurs. [3 marks]

Answer .....

(b) The drug is effective while $C \geqslant 5$. Show that the equation $C = 5$ can be written as $t = 2\ln(2.4t)$. [1 mark]

Answer .....

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(c) Use the iteration $t_{n+1} = 2\ln(2.4t_n)$ with $t_1 = 5$ to find t_2, t_3 and t_4 to 4 decimal places. [3 marks]

Answer .....

(d) By choosing a suitable interval, show that the drug stops being effective at $t = 4.95$, correct to 2 decimal places. [2 marks]

Answer .....

(e) The equation $C = 5$ also has a root near $t = 0.55$. By considering the gradient of $y = 2\ln(2.4t)$, explain why the iteration in part (c) cannot be used to find this root. [1 mark]

Answer .....

Total for X6: 10 marks

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X7 A cup of tea cools in a room where the temperature is $20\,^{\circ}\mathrm{C}$. The temperature of the tea, $\theta\,^{\circ}\mathrm{C}$, is recorded at time t minutes. A graph of $\ln(\theta - 20)$ against t is a straight line passing through the points $(0, 4.1)$ and $(30, 2.9)$.

(a) Explain why the straight-line graph supports a model of the form $\theta - 20 = Ae^{-kt}$, where A and k are positive constants. [2 marks]

Answer .....

(b) Find the value of k, and the value of A to 3 significant figures. [3 marks]

Answer .....

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(c) Find the temperature of the tea when $t = 15$. [2 marks]

Answer .....

(d) Find the time at which the temperature of the tea is $35\,^{\circ}\mathrm{C}$. [2 marks]

Answer .....

(e) State what the model predicts about the temperature of the tea in the long term. [1 mark]

Answer .....

Total for X7: 10 marks

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X8 You may assume that $\sqrt{2}$ is irrational.

(a) Prove by contradiction that if a and b are rational numbers and $a + b\sqrt{2} = 0$, then $b = 0$. [4 marks]

Answer .....

(b) Deduce that $a = 0$, and hence show that if $p + q\sqrt{2} = r + s\sqrt{2}$, where p, q, r and s are rational, then $p = r$ and $q = s$. [3 marks]

Answer .....

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(c) Hence find the positive rational numbers x and y such that $(x + y\sqrt{2})^2 = 17 + 12\sqrt{2}$. [3 marks]

Answer .....

Total for X8: 10 marks

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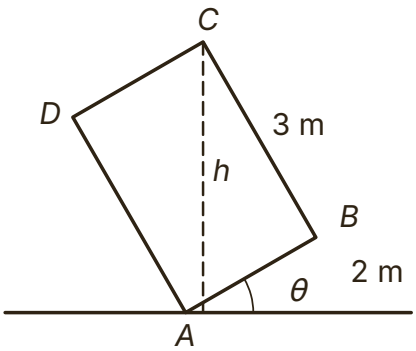
X9 A rectangular crate $ABCD$ has $AB = 2$ m and $BC = 3$ m. The crate is tilted about the corner A, which stays on a horizontal floor. The side AB makes an angle θ radians with the floor, where $0 \leqslant \theta \leqslant \frac{\pi}{2}$.

The height of the corner C above the floor is h metres.



not to scale

(a) Show that $h = 2\sin\theta + 3\cos\theta$. [2 marks]

Answer .....

(b) Express h in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and α to 4 decimal places. [3 marks]

Answer .....

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(c) The crate has to pass under a beam that is 3.5 m above the floor. [4 marks]

Find the set of values of θ for which C is lower than the beam.

Answer .....

(d) Write down the greatest height of C above the floor, and explain what this length is on the crate. [1 mark]

Answer .....

Total for X9: 10 marks

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X10 The curve C has equation

$$x = 3\tan 2y, \qquad -\frac{\pi}{4} < y < \frac{\pi}{4}$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{3}{2\left(9 + x^2\right)}$. [4 marks]

Answer .....

(b) Find $\dfrac{d^2y}{dx^2}$ in terms of x, and hence show that C has a point of inflection at the origin. [4 marks]

Answer .....

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(c) Find an equation of the tangent to C at the point where $y = \frac{\pi}{8}$. [2 marks]

Answer .....

Total for X10: 10 marks

X11

$$f(x) = \frac{5 - x}{(1 + x)(2 - x)}$$

(a) Express $f(x)$ in partial fractions. [3 marks]

Answer .....

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(b) Show that, for small values of x, $f(x) \approx \frac{5}{2} - \frac{7}{4}x + \frac{17}{8}x^2$. [3 marks]

Answer .....

(c) Use the approximation in (b) to estimate $\int_0^{0.2} f(x)\,dx$. Give your answer to 4 decimal places. [2 marks]

Answer .....

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(d) Use integration to find the exact value of $\int_0^{0.2} f(x)\,dx$, giving your answer in the form $\ln k$, and hence find the percentage error in the estimate in (c). [3 marks]

Answer .....

Total for X11: 11 marks

X12 Two chemicals react to form a product. The mass of the product, x grams, at time t minutes is modelled by

$$\frac{dx}{dt} = \frac{(3-x)(5-x)}{8}, \qquad 0 \leqslant x < 3$$

When $t = 0$, $x = 0$.

(a) Express $\dfrac{8}{(3-x)(5-x)}$ in partial fractions. [2 marks]

Answer .....

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(b) Hence show that $x = \dfrac{15\left(e^{\frac{1}{4}t} - 1\right)}{5e^{\frac{1}{4}t} - 3}$. [5 marks]

Answer .....

(c) Find the time at which 2 grams of the product have formed. Give your answer to 3 significant figures. [2 marks]

Answer .....

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(d) Find the limiting value of x as t becomes large, and explain how this value can be seen from the differential equation. [2 marks]

Answer .....

Total for X12: 11 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a complete proof: the assumption, each step with its reason, and a final conclusion.	○	○	○
I can find my own route through a question that has no parts, and give exact answers.	○	○	○
I can set up and solve a model in context, and say what the answer means and where the model stops working.	○	○	○
I can justify a result fully: a range of validity, a nature of a turning point, a root to a stated accuracy.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Which function has derivative $\sec^2 x$? Use it as v.
	Hint 2	You need $\int \tan x \, dx$. Write $\tan x$ as $\dfrac{\sin x}{\cos x}$, or use the formula booklet.
	Hint 3	$\sec \dfrac{\pi}{4} = \sqrt{2}$. Write $\ln \sqrt{2}$ as a multiple of $\ln 2$.
X2	**Hint 1**	The circle touches the y-axis at $(0, 4)$. What does that tell you about the y-coordinate of the centre, and its x-coordinate in terms of the radius?
	Hint 2	Use the fact that P is a distance r from the centre to form an equation in r.
	Hint 3	The tangent at P is perpendicular to the radius to P.
X3	**Hint 1**	Area $= \displaystyle\int y \, \frac{dx}{dt} \, dt$. Find the value of t at which $x = \dfrac{1}{2}$.
	Hint 2	Write $\tan t$ as $\dfrac{\sin t}{\cos t}$. The integrand simplifies to a multiple of $\sin^2 t$.
	Hint 3	Integrate $\sin^2 t$ using $\cos 2t = 1 - 2\sin^2 t$.
X4	**Hint 1**	$fg(x)$ means do g first. Take care: $gf(x)$ has the modulus inside the bracket of g.
	Hint 2	For (b), find where each modulus changes sign. That gives three regions; write the equation without modulus signs in each one.
	Hint 3	A region that gives a false statement such as $3 = 9$ has no solutions. For (c), compare the gradients of the two lines.

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X5	**Hint 1**	The sum keeps growing while the terms being added are positive. Find the last positive term.
	Hint 2	For (b), factorise S_n so that one factor is n.
	Hint 3	For (c), the sum from the 10th term is S_{25} with the first nine terms removed, not the first ten.
X6	**Hint 1**	Differentiate with the product rule and factorise out $e^{-\frac{1}{2}t}$.
	Hint 2	For (b), divide by $12t$, turn the negative power over, then take logarithms.
	Hint 3	For (e), differentiate $2\ln(2.4t)$ and compare its value at $t = 0.55$ with the gradient of $y = t$.
X7	**Hint 1**	Take natural logarithms of both sides of $\theta - 20 = Ae^{-kt}$.
	Hint 2	The gradient of the line is $-k$, not k.
	Hint 3	For (c) and (d), the equation of the line $\ln(\theta - 20) = 4.1 - 0.04t$ is the quickest route.
X8	**Hint 1**	For (a), assume the opposite of what you want: $b \neq 0$. Then you may divide by b.
	Hint 2	For (b), take everything to one side so that the equation has the form (rational) + (rational)$\sqrt{2} = 0$.
	Hint 3	For (c), expand the square and use (b) to get two equations. One of the values of x^2 does not give a rational x: say so.
X9	**Hint 1**	Find the height of B first. Then find how much higher C is than B: what angle does BC make with the vertical?
	Hint 2	For (c), solve the equation $h = 3.5$ first. There are two values of $\theta + \alpha$.
	Hint 3	Decide whether $h < 3.5$ between the two values or outside them, and remember that $0 \leqslant \theta \leqslant \frac{\pi}{2}$.

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X10

Hint 1 Differentiate x with respect to y first. Then use an identity to replace $\sec^2 2y$ by an expression in x.

Hint 2 For (b), differentiate your answer to (a) with respect to x. A zero second derivative is not enough: show the sign changes.

Hint 3 For (c), find x when $y = \dfrac{\pi}{8}$ and use the formula from (a).

X11

Hint 1 For (b), expand each partial fraction separately; $(2-x)^{-1}$ needs a factor of $\dfrac{1}{2}$ taken out.

Hint 2 For (d), $\displaystyle\int \frac{1}{2-x}\,dx = -\ln(2-x)$. Combine the logarithms into one.

Hint 3 Percentage error is (estimate $-$ exact) $\div$ exact $\times\, 100$.

X12

Hint 1 Take care with the signs: integrating $\dfrac{4}{3-x}$ gives $-4\ln(3-x)$.

Hint 2 Find the constant, then write the two logarithms as one before removing them.

Hint 3 For (d), divide the top and the bottom of the fraction by $e^{\frac{1}{4}t}$ before letting t become large.

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