A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 1: ALGEBRAIC EXPRESSIONS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Algebraic expressions: A* questions

74 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 Find all the real values of x for which [9 marks]

$$\left(x^2 - 5x + 5\right)^{x^2 - 9x + 20} = 1.$$

You must show your working.

Answer $x = \ldots$

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X2 In this question n is an integer.

(a) Write $n^5 - n$ as a product of four factors, three of which are linear. [2 marks]

Answer .....

(b) Explain why $n^5 - n$ is a multiple of 6. [2 marks]

(c) By considering the cases $n = 5k$, $n = 5k \pm 1$ and $n = 5k \pm 2$, where k is an integer, prove that $n^5 - n$ is a multiple of 5. [4 marks]

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(d) Hence prove that $n^5 - n$ is a multiple of 30 for every integer n, and find the largest integer that divides $n^5 - n$ for every integer n. [3 marks]

Answer Largest integer $= \ldots$

Total for X2: 11 marks

X3 In this question n is a positive integer. Do not use a calculator.

(a) Show that $\sqrt{n+1} - \sqrt{n} = \dfrac{1}{\sqrt{n+1} + \sqrt{n}}$. [2 marks]

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(b) Hence prove that [4 marks]

$$2\left(\sqrt{n+1}-\sqrt{n}\right) < \frac{1}{\sqrt{n}} < 2\left(\sqrt{n}-\sqrt{n-1}\right).$$

(c) $S = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{100}}$. [5 marks]

Use part (b) to prove that $18 < S < 19$.

Total for X3: 11 marks

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X4 The numbers a, b and c are non-zero and $2^a = 3^b = 6^c$.

(a) Prove that $c = \dfrac{ab}{a+b}$. [5 marks]

(b) Given that $c = 1$, show that $(a-1)(b-1) = 1$. [2 marks]

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(c) Given that $c = 1$, prove that $a + b > 4$. [4 marks]

Total for X4: 11 marks

X5 Do not use a calculator in this question.

(a) Show that $\left(2 + \sqrt{3}\right)^{-1} = 2 - \sqrt{3}$. [2 marks]

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(b) Solve $\left(2+\sqrt{3}\right)^{x}+\left(2-\sqrt{3}\right)^{x}=14.$ [5 marks]

Answer $x = \ldots$

(c) Find the set of values of the constant k for which the equation $\left(2+\sqrt{3}\right)^{x}+\left(2-\sqrt{3}\right)^{x}=k$ has exactly two real solutions. [4 marks]

Answer k such that $\ldots$

Total for X5: 11 marks

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X6 Without using a calculator, find the largest integer that is less than $\left(\sqrt{3}+\sqrt{2}\right)^6$. You must show your working. [8 marks]

Answer Largest integer $=\ldots$

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X7 A model for the energy used by a resting mammal is

$$R = 70m^{\frac{3}{4}},$$

where R is the energy used in kilocalories per day and m is the mass of the mammal in kilograms. Do not use a calculator in this question.

(a) Find the value of R given by the model for a mammal of mass 81 kg. [2 marks]

Answer $R = \ldots$

(b) Mammal A uses energy at 8 times the rate of mammal B. Use the model to find how many times heavier A is than B. [3 marks]

Answer ... times

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(c) The energy used per kilogram of body mass is $P = \dfrac{R}{m}$. Show that $P = 70m^{-\frac{1}{4}}$. Mammal C has 625 times the mass of mammal D. Find the value of $\dfrac{P_C}{P_D}$. [4 marks]

Answer $\dfrac{P_C}{P_D} = \ldots$

(d) A dog of mass 16 kg is measured to use 700 kilocalories per day at rest. Find the value given by the model, state whether the model gives too high or too low a value for this dog, and give one reason why the model might not fit a particular animal. [4 marks]

Total for X7: 13 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can solve equations where the unknown is in the index by putting t equal to a power.	○	○	○
I can use conjugate surds to rationalise, to bound expressions and to find exact values.	○	○	○
I can factorise expressions in n to prove a divisibility result.	○	○	○
I can use index laws in a model and compare two cases by dividing.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 A power a^b equals 1 when $a = 1$, or when $b = 0$ (with $a \neq 0$), or when $a = -1$ and b is even.

Hint 2 Each case gives a quadratic in x. Solve all three.

Hint 3 In the last two cases, check each answer: the base must not be 0, and with base -1 the index must be even.

X2

Hint 1 $n^5 - n = n(n^4 - 1)$, and $n^4 - 1$ is a difference of two squares.

Hint 2 For (b), look at the three linear factors together. For (c), only $n^2 + 1$ is needed when $n = 5k \pm 2$.

Hint 3 Being a multiple of 6 and of 5 makes it a multiple of 30. To show nothing larger works, find a value of n for which $n^5 - n = 30$.

X3

Hint 1 Multiply $\sqrt{n+1} - \sqrt{n}$ by $\sqrt{n+1} + \sqrt{n}$.

Hint 2 Use (a) to turn each bracket into a fraction. Then compare $\sqrt{n+1} + \sqrt{n}$ and $\sqrt{n} + \sqrt{n-1}$ with $2\sqrt{n}$.

Hint 3 Write the left inequality for $n = 1, 2, 3, \ldots$ under each other and add: almost every term cancels. For the upper bound start at $n = 2$, because $\sqrt{0} = 0$ would give a weaker result.

X4

Hint 1 Call the common value k. Then $2 = k^{\frac{1}{a}}$. Write 3 and 6 the same way.

Hint 2 For (b), put $c = 1$ into the result of (a) and multiply out $(a-1)(b-1)$.

Hint 3 For (c), start from $(a-b)^2 \geq 0$. Explain why a and b are positive and why they cannot be equal.

X5

Hint 1 Multiply top and bottom of $\dfrac{1}{2+\sqrt{3}}$ by $2 - \sqrt{3}$.

Hint 2 Let $t = \left(2+\sqrt{3}\right)^x$. Part (a) tells you what $\left(2-\sqrt{3}\right)^x$ is in terms of t.

Hint 3 t is always positive, and each positive t gives one x. You need the quadratic in t to have two different positive roots.

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X6

Hint 1 Work with $u = \sqrt{3} + \sqrt{2}$ and its partner $v = \sqrt{3} - \sqrt{2}$. Find u^2 and v^2 first.

Hint 2 $u^6 + v^6 = (u^2)^3 + (v^2)^3$. When you add the two expansions, the terms with $\sqrt{6}$ cancel, so the sum is a whole number.

Hint 3 $v = \dfrac{1}{\sqrt{3} + \sqrt{2}}$, so v^6 is between 0 and 1. Use this to place u^6 between two whole numbers.

X7

Hint 1 $m^{\frac{3}{4}} = \left(m^{\frac{1}{4}}\right)^3$. Find the fourth root first.

Hint 2 Divide the two equations: the 70 cancels and you get $\left(\dfrac{m_A}{m_B}\right)^{\frac{3}{4}} = 8$.

Hint 3 $\dfrac{m^{\frac{3}{4}}}{m} = m^{\frac{3}{4}-1}$. For (d), $16^{\frac{1}{4}} = 2$.

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