A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 10: TRIGONOMETRIC IDENTITIES AND EQUATIONS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Trigonometric identities and equations: A* questions

74 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 Solve, for $-180° < \theta \leq 180°$, the equation $2\sin^3\theta + 3\cos^2\theta = 2 + \sin\theta$ [9 marks]. Give your answers to 1 decimal place where they are not exact.

Answer $\theta = \ldots$

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X2

(a) Given that $\sin\theta \neq 0$ and $\cos\theta \neq 0$, prove that $\tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{1}{\sin\theta\cos\theta}$. [3 marks]

(b) Solve, for $0° < \theta < 360°$, the equation $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{4}{\sqrt{3}}$. [4 marks]

Answer $\theta = \ldots$

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(c) Kai says: "The equation $\tan\theta + \dfrac{1}{\tan\theta} = 1.5$ has a solution between $0°$ and $360°$." Show that Kai is wrong. [2 marks]

Total for X2: 9 marks

X3 The angle θ is such that $\cos\theta \neq 0$.

(a) Prove that $\dfrac{\tan^2\theta}{1+\tan^2\theta} \equiv \sin^2\theta$. [3 marks]

(b) Hence solve, for $0° \leq \theta < 360°$, the equation $\dfrac{2\tan^2\theta}{1+\tan^2\theta} = 3\cos\theta$. [4 marks]

Answer $\theta = \ldots$

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(c) Hence solve, for $0^\circ \leq x < 360^\circ$, the equation $\dfrac{2\tan^2 2x}{1+\tan^2 2x} = 3\cos 2x$. [3 marks]

Answer $x = \ldots$

Total for X3: 10 marks

X4 Find the set of values of x, with $-180^\circ < x < 180^\circ$ and $x \neq \pm 90^\circ$, for which $2\sin x > \tan x$. Give your answer in set notation. [8 marks]

Answer $\{x : \ldots\}$

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X5 The angles x and y satisfy the simultaneous equations $\sin x + \cos y = 1$ and $\sin^2 x + \cos^2 y = \frac{5}{8}$.

(a) Show that $\sin x \cos y = \frac{3}{16}$. [2 marks]

(b) Find the two possible pairs of values of $\sin x$ and $\cos y$. [3 marks]

Answer $\sin x = \ldots$, $\cos y = \ldots$ or $\sin x = \ldots$, $\cos y = \ldots$

(c) Given that $0° \leq x < 360°$ and $0° \leq y < 360°$, find how many pairs (x, y) satisfy both equations. Find the pairs in which x and y are both acute, to 1 decimal place. [3 marks]

Answer number of pairs $= \ldots$; acute pairs: $\ldots$

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(d) The second equation is changed to $\sin^2 x + \cos^2 y = k$, and the first is kept. Find the least value of k for which the two equations have a solution. [2 marks]

Answer $k = \ldots$

Total for X5: 10 marks

X6 The angles x and y are acute, with $\sin x = 2\sin y$ and $\tan x = 3\tan y$. [8 marks]

Show that $\cos y = \dfrac{3\sqrt{6}}{8}$, and find x and y to 1 decimal place.

Answer $x = \ldots$, $y = \ldots$

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X7 In this question, give exact answers.

(a) Solve, for $0^\circ \leq x < 360^\circ$, the equation $9^{\cos x} - 4 \times 3^{\cos x} + 3 = 0$. [4 marks]

Answer $x = \ldots$

(b) Solve, for $0^\circ < \theta < 360^\circ$, the equation $\log_2(\sin\theta) + \log_2(4\cos\theta) = 0$. [6 marks]

Answer $\theta = \ldots$

Total for X7: 10 marks

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X8

(a) Prove that $(1 + \sin\theta + \cos\theta)^2 \equiv 2(1 + \sin\theta)(1 + \cos\theta)$. [3 marks]

(b) Hence solve, for $0° \leq \theta < 360°$, the equation $(1 + \sin\theta + \cos\theta)^2 = 3(1 + \cos\theta)$. Explain why dividing both sides by $1 + \cos\theta$ would lose a solution. [4 marks]

Answer $\theta = \ldots$

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(c) By considering $(\sin\theta - \cos\theta)^2$, show that $\sin\theta + \cos\theta \leq \sqrt{2}$, and hence that $(1 + \sin\theta + \cos\theta)^2 \leq 3 + 2\sqrt{2}$ for all θ. [3 marks]

Total for X8: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can turn a trigonometric equation into a quadratic or a cubic and solve it, rejecting values outside -1 to 1.	○	○	○
I can prove an identity and use it to solve an equation without losing solutions.	○	○	○
I can solve a trigonometric inequality by looking at the sign of each factor.	○	○	○
I can link trigonometry with indices, logarithms and simultaneous equations.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Write everything in terms of $\sin\theta$. You get a cubic in $\sin\theta$.
	Hint 2	Try $\sin\theta = \dfrac{1}{2}$ in the cubic. If it works, $(2\sin\theta - 1)$ is a factor.
	Hint 3	A value of $\sin\theta$ outside $-1 \leq \sin\theta \leq 1$ gives no angles. For a negative sine, the two angles in the range are both negative.
X2	**Hint 1**	Write $\tan\theta$ as $\dfrac{\sin\theta}{\cos\theta}$ and put the two fractions over one denominator.
	Hint 2	Let $t = \tan\theta$ and multiply through by t. You get a quadratic in t.
	Hint 3	A quadratic with a negative discriminant has no real roots.
X3	**Hint 1**	Replace $\tan^2\theta$ by $\dfrac{\sin^2\theta}{\cos^2\theta}$, then multiply the top and the bottom by $\cos^2\theta$.
	Hint 2	After using part (a), write $\sin^2\theta$ as $1 - \cos^2\theta$. You get a quadratic in $\cos\theta$.
	Hint 3	If $0° \leq x < 360°$, then $2x$ runs from $0°$ up to $720°$. Go round twice.
X4	**Hint 1**	Write $\tan x$ as $\dfrac{\sin x}{\cos x}$ and bring everything to one side.
	Hint 2	Multiplying an inequality by $\cos x$ is not safe, as $\cos x$ can be negative. Multiply by $\cos^2 x$ instead, or keep the fraction.
	Hint 3	Mark where $\sin x = 0$, $\cos x = 0$ and $\cos x = \dfrac{1}{2}$, then find the sign of each factor on each interval.
X5	**Hint 1**	Square $\sin x + \cos y = 1$ and use the second equation.
	Hint 2	Two numbers with a known sum and product are the roots of a quadratic: $t^2 - (\text{sum})t + \text{product} = 0$.
	Hint 3	For part (d), put $\cos y = 1 - \sin x$ and complete the square in $\sin x$.
X6	**Hint 1**	Write $\tan x = 3\tan y$ with sines and cosines, then replace $\sin x$ by $2\sin y$.
	Hint 2	You should find $\cos x$ in terms of $\cos y$. Now use $\sin^2 x + \cos^2 x = 1$.
	Hint 3	Write everything in terms of $\cos y$, using $\sin^2 y = 1 - \cos^2 y$.

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X7

Hint 1 $9^{\cos x}$ is the square of $3^{\cos x}$. Let $u = 3^{\cos x}$.

Hint 2 Add the logarithms first, then remove the logarithm. Check which quadrants are allowed: you cannot take the logarithm of a negative number.

Hint 3 From $\sin\theta\cos\theta = \dfrac{1}{4}$, square both sides and write $\cos^2\theta$ as $1 - \sin^2\theta$. You get a quadratic in $\sin^2\theta$.

X8

Hint 1 Expand the square: there are six terms. Two of them add to 1.

Hint 2 Use part (a), then take out the common factor $(1 + \cos\theta)$ instead of dividing by it.

Hint 3 Expand $(\sin\theta - \cos\theta)^2 \geq 0$ to bound $2\sin\theta\cos\theta$, then expand $(\sin\theta + \cos\theta)^2$.

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