A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 12: DIFFERENTIATION · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-year-1-chapter-12-a-star/

Differentiation: A* questions

78 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The function f is defined by $f(x) = \dfrac{2x}{x+3}$, $x \neq -3$.

(a) Prove, from first principles, that $f'(x) = \dfrac{6}{(x+3)^2}$. [5 marks]

(b) The line l has equation $2x - 3y = 7$. Find the equations of the two tangents to the curve $y = f(x)$ that are parallel to l. [5 marks]

Answer tangents: ...

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(c) Jay says: "$f'(x) > 0$ for every $x \neq -3$, so $f(-4) < f(0)$." Show that Jay is wrong, and explain the mistake in his reasoning. [3 marks]

Total for X1: 13 marks

X2 The curve C has equation $y = x^2 + 4$ and the line l has equation $y = 2x - 3$.

(a) Show that C and l do not meet. [2 marks]

(b) The tangent to C at the point P is parallel to l. Find the coordinates of P and an equation of this tangent. [3 marks]

Answer $P = \ldots$, tangent: $\ldots$

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(c) The normal to C at P meets l at Q. Show that $PQ = \frac{6\sqrt{5}}{5}$. [4 marks]

(d) Prove that no point of C is closer than $\frac{6\sqrt{5}}{5}$ to any point of l. [3 marks]

Total for X2: 12 marks

X3 The curves C_1 and C_2 have equations $y = x^2 + 1$ and $y = 6x - x^2 - 4$.

(a) Show that C_1 and C_2 do not meet. [2 marks]

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(b) A line is a tangent to both C_1 and C_2. Find the equations of the two lines with this property. [6 marks]

Answer lines: . . .

(c) The two lines meet at T. Find the coordinates of T, and show that T is the midpoint of the two points at which the line with the smaller gradient touches C_1 and C_2. [3 marks]

Total for X3: 11 marks

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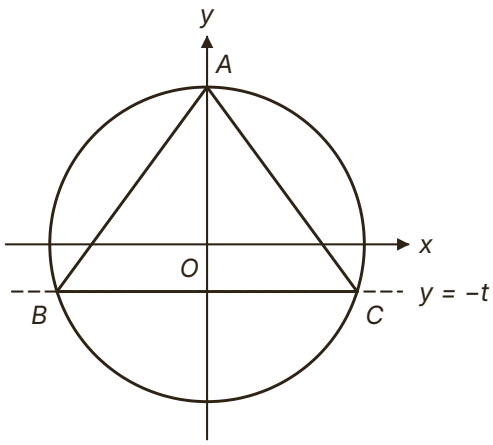
X4 The circle with equation $x^2 + y^2 = 25$ passes through $A(0, 5)$. The points B and C lie on the circle and on the line $y = -t$, where $-5 < t < 5$, with B to the left of C. The diagram shows one position. Let S be the area of triangle ABC.

(a) Show that $S^2 = (5 + t)^2(25 - t^2)$. [3 marks]



(b) Explain why S is greatest when S^2 is greatest. Hence find the value of t for which S is greatest, and show that it gives a maximum. [5 marks]

Answer $t = \ldots$

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(c) Find the exact greatest area of triangle ABC, and show that the triangle is then equilateral. [3 marks]

Answer greatest area $= \ldots$

Total for X4: 11 marks

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X5 The curve C has equation $y = \frac{1}{4}x^2$. The point K is $(0, k)$, where k is a constant. Find, in terms of k, the number of normals to C that pass through K, justifying your answer. In the case $k = 6$, find equations of these normals, and the area of the triangle whose vertices are the points of C at which they are drawn. [10 marks]

Answer number of normals: ... ; normals: ... ; area = ...

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X6 The points $A(a, a^2)$ and $B(b, b^2)$, where $a \neq b$, lie on the curve $y = x^2$. The chord AB passes through the point $(0, k)$, where $k > 0$. The tangents to the curve at A and at B meet at T. Prove that T lies on the line $y = -k$. Find the value of k for which the tangents at A and B are perpendicular for every chord through $(0, k)$. [10 marks]

Answer $k = \ldots$

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X7 $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$, for all real x, where $x^{\frac{1}{3}}$ means the real cube root of x.

(a) Show that $f'(x) = \dfrac{5(x-2)}{3x^{\frac{1}{3}}}$ for $x \neq 0$. [3 marks]

(b) Find the exact coordinates of the stationary point of the curve $y = f(x)$, and show that it is a minimum. [3 marks]

Answer ...

(c) Find the set of values of x for which f is decreasing. [3 marks]

Answer ...

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(d) $f'(0)$ does not exist. Show that $(0, 0)$ is nevertheless a local maximum of the curve. [2 marks]

Total for X7: 11 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can differentiate a fraction from first principles, showing the limit clearly.	○	○	○
I can find a line that is a tangent to two curves, or the normals to a curve through a given point.	○	○	○
I can use tangents and normals to prove facts about curves and the lines near them.	○	○	○
I can find and justify greatest values, including by maximising a square, and explain stationary points where the derivative does not exist.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Write $f(x+h)-f(x)$ as one fraction. Almost everything in the numerator cancels.
	Hint 2	Parallel lines have equal gradients. Make y the subject of $2x-3y=7$ to read off its gradient, then solve $f'(x) =$ that gradient.
	Hint 3	Work out $f(-4)$. "Increasing" compares two points only when the curve runs unbroken between them. What happens at $x=-3$?
X2	**Hint 1**	Two graphs meet where their y values are equal. Use the discriminant of the quadratic you get.
	Hint 2	The normal at P is perpendicular to the tangent at P, and so it is also perpendicular to l.
	Hint 3	Compare C with its tangent at P: subtract and complete the square. Then think of the region between two parallel lines.
X3	**Hint 1**	Set the two expressions for y equal and look at the discriminant.
	Hint 2	Write the tangent to C_1 at $x=a$ and the tangent to C_2 at $x=b$, each in the form $y=mx+c$. They are the same line when both m and c agree.
	Hint 3	From the gradients, $b=3-a$. Put this into the equation for the intercepts to get a quadratic in a.
X4	**Hint 1**	Put $y=-t$ into the equation of the circle to find the x-coordinates of B and C. The base is BC and the height is the vertical distance from A to the line $y=-t$.
	Hint 2	S itself involves a square root, which you cannot yet differentiate as a product. S^2 is a polynomial in t, and it is greatest exactly when S is.
	Hint 3	$t=-5$ is a root of the derivative of S^2 that is outside the range. Use the factor theorem to find the other factor.
X5	**Hint 1**	Take a general point $\left(a, \frac{1}{4}a^2\right)$ on C and write the equation of the normal there. Treat $a=0$ on its own: the tangent there is horizontal.
	Hint 2	Put $x=0$ into the normal to find where it crosses the y-axis. Ask for which values of k that height can be k.
	Hint 3	$a^2=4(k-2)$ has two non-zero solutions, one, or none. Do not forget the normal at the origin.

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X6

Hint 1 Find the gradient of the chord AB and factorise $b^2 - a^2$. Then use the fact that the chord passes through $(0, k)$.

Hint 2 The tangent at (a, a^2) has gradient $2a$. Write both tangents and solve them together; you will need $a \neq b$.

Hint 3 Two lines are perpendicular when the product of their gradients is -1. Compare this with the relation between a and b you found from the chord.

X7

Hint 1 Differentiate each power of x as usual, then take out the factor $\frac{5}{3}x^{-\frac{1}{3}}$.

Hint 2 For $f(2)$, write $2^{\frac{5}{3}} = 2 \times 2^{\frac{2}{3}}$. For the nature, use f'' or the sign of f' either side of $x = 2$.

Hint 3 The cube root of a negative number is negative, so check the sign of f' on $x < 0$ as well. Near $x = 0$, factorise $f(x)$ and look at the sign of each factor.

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