A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 13: INTEGRATION · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-year-1-chapter-13-a-star/

Integration: A* questions

83 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The curve C has equation $y = \sqrt{x}\,(x - 4)$, $x \geq 0$.

(a) Show that $\displaystyle\int_0^9 \sqrt{x}\,(x-4)\,dx = \frac{126}{5}$. [4 marks]

(b) Find the exact total area of the regions bounded by C, the x-axis and the line $x = 9$. [5 marks]

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(c) Find the value of a, where $a > 4$, for which $\int_0^a \sqrt{x}\,(x - 4)\,dx = 0$, and explain what this value of a means for the regions between C and the x-axis. [4 marks]

Total for X1: 13 marks

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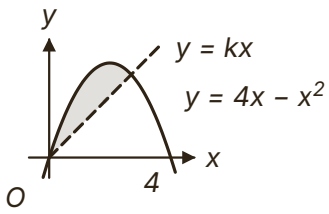
X2 The region R is bounded by the curve $y = 4x - x^2$ and the x-axis. The line $y = kx$, where $0 < k < 4$, divides R into two parts. The part between the line and the curve has $\frac{1}{8}$ of the area of R. Find the value of k. The diagram is not drawn to scale. [8 marks]



Answer $k = \ldots$

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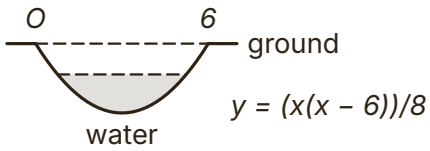
X3 A straight channel is 200 m long. Its cross-section is modelled by the curve $y = \dfrac{x(x-6)}{8}$, $0 \leq x \leq 6$, where x and y are in metres and the ground is the line $y = 0$.

(a) Find the area of the cross-section of the channel. [4 marks]



(b) The surface of the water is 0.5 m below the ground. Show that the water surface meets the sides of the channel where $x = 3 \pm \sqrt{5}$. [2 marks]

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(c) Find the volume of water in the channel. Give your answer in the form $\frac{a\sqrt{5}}{b}$ m³, where a and b are integers, and also to 3 significant figures. [6 marks]

(d) Give two reasons why the model may not give the true volume of water. [2 marks]

Total for X3: 14 marks

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X4 The curve C has equation $y = x^2 + 9$. Two tangents to C pass through the origin O.

(a) Show that the tangent to C at the point where $x = p$ has equation $y = 2px - p^2 + 9$. [3 marks]

(b) Hence find the equations of the two tangents that pass through O. [3 marks]

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(c) Find the area of the region bounded by C and the two tangents. [5 marks]

(d) Show that, for the curve $y = x^2 + c$, where $c > 0$, the region bounded by the curve and the two tangents through O has area $\frac{2}{3}c^{\frac{3}{2}}$. [3 marks]

Total for X4: 14 marks

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X5 n is an integer with $n \geq 2$. The curves $y = x^n$ and $y = x^{\frac{1}{n}}$, $x \geq 0$, meet at the origin and at $(1, 1)$. Show that the area of the region between the two curves is $\dfrac{n-1}{n+1}$, and find the smallest value of n for which this area is greater than 0.9. [8 marks]

Answer $n = \ldots$

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X6 A curve $y = f(x)$, $x > 0$, has $f'(x) = \dfrac{x^2 - 4}{x^{\frac{3}{2}}}$ and passes through the point $(1, 10)$.

(a) Find $f(x)$. [4 marks]

(b) Show that the curve has exactly one stationary point, and find its exact coordinates. [4 marks]

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(c) Use $f''(x)$ to show that this stationary point is a minimum. [2 marks]

(d) Hence state the range of f. [2 marks]

Total for X6: 12 marks

X7 The curves $y = \dfrac{16}{x^2}$ and $y = 17 - x^2$, $x > 0$, meet at the points A and B, where A is nearer to the y-axis.

(a) Find the coordinates of A and B. [4 marks]

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(b) Find the area of the region between the two curves. [5 marks]

(c) The straight line through A and B divides this region into two parts. Show that the areas of the two parts are in the ratio $1 : 3$. [5 marks]

Total for X7: 14 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find areas of regions below the x-axis and regions made of several parts.	○	○	○
I can find the area between a curve and a line, including when the line depends on a constant.	○	○	○
I can integrate fractional and negative powers and give exact answers.	○	○	○
I can use integration and differentiation together to find a curve and its least value.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Write $\sqrt{x}\,(x-4)$ as two powers of x before you integrate.

Hint 2 Part of the area is below the x-axis, where the integral is negative. Find where C crosses the axis and integrate each part on its own.

Hint 3 For (c), take out the common factor $a^{\frac{3}{2}}$. An integral of 0 means the area below and the area above cancel.

X2

Hint 1 Find where the line meets the curve, in terms of k.

Hint 2 The area between the line and the curve is the integral of (curve $-$ line) between those two points. Simplify it to a power of $(4-k)$.

Hint 3 Find the area of R, then set the first area equal to $\frac{1}{8}$ of it.

X3

Hint 1 An integral of a region below the x-axis is negative. The area is its size.

Hint 2 For the water, integrate (water line $-$ curve), which is $-0.5 - \frac{x(x-6)}{8}$, between the two points from (b).

Hint 3 The limits are symmetric about $x=3$. Writing $u = x-3$ makes the surds simpler. Then multiply by the length.

X4

Hint 1 The tangent at $x=p$ passes through (p, p^2+9) with gradient $2p$.

Hint 2 Substitute $x=0$, $y=0$ into the tangent from (a) to find p.

Hint 3 Curve minus tangent is a perfect square, $(x-3)^2$. Use the symmetry of the region about the y-axis.

X5

Hint 1 Which curve is higher for $0<x<1$? Try $x=\frac{1}{4}$ with $n=2$.

Hint 2 $\int x^{\frac{1}{n}}\,dx = \frac{x^{\frac{1}{n}+1}}{\frac{1}{n}+1}$. Simplify $\frac{1}{n}+1$ to $\frac{n+1}{n}$.

Hint 3 $n+1>0$, so you can multiply both sides of the inequality by it. Check $n=19$ itself.

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X6

Hint 1 Divide each term of $x^2 - 4$ by $x^{\frac{3}{2}}$ before integrating. Then use $(1, 10)$ to find the constant.

Hint 2 $x^{\frac{3}{2}} > 0$, so $f'(x) = 0$ only when $x^2 - 4 = 0$. Remember that $x > 0$.

Hint 3 $\dfrac{8}{\sqrt{2}} = 4\sqrt{2}$. For (d), there is only one stationary point and it is a minimum.

X7

Hint 1 Multiply through by x^2. The equation is a quadratic in x^2.

Hint 2 $\dfrac{16}{x^2} = 16x^{-2}$. Check which curve is on top between A and B by trying $x = 2$.

Hint 3 Find the area between the line and the parabola, then subtract it from your answer to (b).

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