A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 14: EXPONENTIALS AND LOGARITHMS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Exponentials and logarithms: A* questions

78 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The acidity of a solution is measured by its pH, where $\mathrm{pH} = -\log_{10} h$ and h is the concentration of hydrogen ions in moles per litre. When two solutions are mixed, the value of h for the mixture is the mean of their values of h, weighted by their volumes.

(a) A solution has pH 2.3. Find h for this solution. [2 marks]

(b) Equal volumes of a solution of pH 2 and a solution of pH 4 are mixed. Ali says that the mixture has pH 3. Show that Ali is wrong, and find the pH of the mixture to 2 decimal places. [4 marks]

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(c) Show that when equal volumes of solutions of pH p and pH $p + 2$ are mixed, the pH of the mixture is $p + \log_{10} \frac{200}{101}$. [3 marks]

(d) A volume V of a solution of pH 3 is mixed with a volume nV of pure water, which has pH 7. The mixture has pH 4. Find the value of n to 3 significant figures. [4 marks]

Total for X1: 13 marks

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X2 At the start of a year, Kemi puts $£2000$ into an account that pays 4% interest each year, and Lewis puts $£1200$ into an account that pays 7% interest each year. The interest is added at the end of each year and no money is taken out. Find the number of complete years after which Lewis's account is first worth more than Kemi's. You must show your working. [8 marks]

Answer ... years

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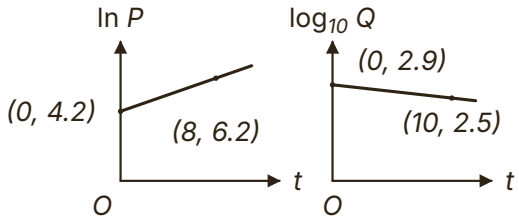
X3 The numbers of two species of fish in a lake, P and Q, are recorded each year, t years after the start of a study. A graph of $\ln P$ against t is a straight line through $(0, 4.2)$ and $(8, 6.2)$. A graph of $\log_{10} Q$ against t is a straight line through $(0, 2.9)$ and $(10, 2.5)$.

(a) Write P in the form Ae^{kt} and Q in the form Bc^t, giving A, k, B and c to 3 significant figures. [4 marks]



(b) Find the value of t when the two species have equal numbers. Give your answer to 3 significant figures. [4 marks]

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(c) Find the rate at which P is increasing at this time, in fish per year. [3 marks]

(d) Give two reasons why the model for P is not suitable for large values of t. [2 marks]

Total for X3: 13 marks

X4 In this question $\log$ means $\log_{10}$. A positive whole number N has d digits when $10^{d-1} \leq N < 10^{d}$.

(a) Find the number of digits in 2^{2026}. [3 marks]

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(b) Find the first digit of 2^{2026}. [3 marks]

(c) n is a positive integer. 2^n has a digits and 5^n has b digits. Prove that $a + b = n + 1$. [5 marks]

Total for X4: 11 marks

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X5 Solve the simultaneous equations [8 marks]

$$2^x \times 3^y = 10, \qquad 3^x \times 2^y = 15.$$

Give your answers in the form $\log_6 k$, where k is an integer.

Answer $x = \ldots$, $y = \ldots$

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X6 Light shining down into a lake gets weaker with depth. The intensity at depth d metres is modelled by $I = I_0 e^{-kd}$, where I_0 is the intensity at the surface and k is a constant. At a depth of 2 m, 70% of the light remains.

(a) Show that $k = \frac{1}{2}\ln\frac{10}{7}$, and find k to 3 significant figures. [3 marks]

(b) Mia says: "30% of the light is lost in the first 2 m, so 40% remains at a depth of 4 m." Show that Mia is wrong. [3 marks]

(c) Find the depth at which 1% of the light remains. [3 marks]

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(d) In a different lake the model holds down to a depth of 10 m. Below 10 m the water is cloudier, and the intensity is modelled by $I = I_{10}e^{-3k(d-10)}$, where I_{10} is the intensity at 10 m and k is the value from (a). Find the depth at which 1% of the surface light remains in this lake. [4 marks]

Total for X6: 13 marks

X7 In this question a, b and x are positive numbers, none of which is equal to 1.

(a) Prove that $\log_a b \times \log_b a = 1$. [3 marks]

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(b) Solve $\log_3 x = 4 \log_x 3$. [4 marks]

(c) Find the set of values of the constant k for which the equation $\log_3 x + k \log_x 3 = 3$ has two different solutions. [5 marks]

Total for X7: 12 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can turn a straight-line graph of $\ln y$ or $\log_{10} y$ into an exponential model.	○	○	○
I can solve exponential equations and inequalities with logarithms and check a whole-number answer.	○	○	○
I can use the laws of logarithms in proofs and in equations with a parameter.	○	○	○
I can use and criticise an exponential model in context.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Change each pH back to a value of h with $h = 10^{-\mathrm{pH}}$ before you take any mean.

Hint 2 For (c), take 10^{-p} out of both terms, then use the laws of logarithms.

Hint 3 For (d), the weighted mean is (total amount) $\div$ (total volume): $\dfrac{V \times 10^{-3} + nV \times 10^{-7}}{V + nV}$.

X2

Hint 1 Write each account's value after n years as a starting amount times a multiplier to the power n.

Hint 2 Divide both sides by $1200(1.04)^n$ so that n appears once, in $\left(\dfrac{1.07}{1.04}\right)^n$.

Hint 3 Take logarithms and solve for n. n is a whole number, so check the two whole numbers either side.

X3

Hint 1 For each line find the gradient and the intercept, then write $\ln P = \ldots$ and $\log_{10} Q = \ldots$ before removing the logarithms.

Hint 2 $\ln Q$ and $\log_{10} Q$ are different. Use $\ln Q = \log_{10} Q \times \ln 10$, or set the two models equal and take $\ln$ of both sides.

Hint 3 The derivative of e^{kt} is ke^{kt}, so $\dfrac{dP}{dt} = kP$.

X4

Hint 1 $\log N$ tells you the number of digits: if $\log N = 609.9$, then N is between 10^{609} and 10^{610}.

Hint 2 Split $10^{609.887}$ into $10^{0.887} \times 10^{609}$. The first part gives the leading digits.

Hint 3 $2^n \times 5^n = 10^n$. Multiply the two digit inequalities. Why can neither number be exactly a power of 10?

X5

Hint 1 Multiply the two equations: $2^x \times 3^x = 6^x$.

Hint 2 Now divide one equation by the other. This gives a second equation in x and y.

Hint 3 You have $x + y$ and $x - y$. Write 1 as $\log_6 6$ and $\dfrac{1}{2}\log_6 900$ as $\log_6 30$.

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X6

Hint 1 At $d = 2$, $\dfrac{I}{I_0} = 0.7$. Take $\ln$ of both sides.

Hint 2 $e^{-4k} = \left(e^{-2k}\right)^2$. Compare with Mia's figure.

Hint 3 For (d), first find the fraction left at 10 m. Then the cloudier water has to reduce that fraction to 0.01.

X7

Hint 1 Write $\log_a b = p$ as an index statement, $a^p = b$, and take logarithms to base b.

Hint 2 Put $u = \log_3 x$. By (a), $\log_x 3 = \dfrac{1}{u}$.

Hint 3 Each value of u gives one value of x, but $u = 0$ would mean $x = 1$. Check what happens when $k = 0$.

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