A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 2: QUADRATICS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Quadratics: A* questions

80 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 [9 marks]

$$f(x) = \frac{2x+1}{x^2+2}, \qquad x \in \mathbb{R}.$$

By writing $y = f(x)$ and forming a quadratic equation in x, find the range of f. Find the value of x at which $f(x)$ is greatest and the value of x at which it is least.

Answer Range: ... Greatest at $x = \ldots$ Least at $x = \ldots$

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X2 The curve C has equation $y = x^2$.

(a) Show that the line $y = mx + c$ is a tangent to C if and only if $c = -\frac{m^2}{4}$, and find, in terms of m, the coordinates of the point of contact. [4 marks]

(b) Two tangents to C have gradients m_1 and m_2, where $m_1 \neq m_2$. Show that they meet at the point $\left(\frac{m_1 + m_2}{4}, \frac{m_1 m_2}{4}\right)$. [3 marks]

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(c) Prove that whenever two tangents to C are perpendicular, they meet on a fixed line, and state the equation of this line. [2 marks]

Answer $y = \ldots$

(d) Two perpendicular tangents to C meet at the point with x-coordinate $\frac{3}{8}$. Find the equations of the two tangents. [4 marks]

Total for X2: 13 marks

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X3 In this question a right-angled triangle has shorter sides a and b and hypotenuse c, all in cm.

(a) A right-angled triangle has perimeter 40 cm and area 60 cm². [4 marks]
Using $(a+b)^2 = a^2 + b^2 + 2ab$, find c, and hence find a and b.

Answer $c = \ldots$, a and $b = \ldots$

(b) Prove that no right-angled triangle has perimeter 12 cm and area 9 cm². [4 marks]

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(c) A right-angled triangle has perimeter 12 cm and area A cm². [5 marks]

Show that $c = 6 - \dfrac{A}{6}$. Hence find, in the form $p - q\sqrt{2}$ where p and q are integers, the largest possible value of A.

Answer Largest $A = \ldots$

Total for X3: 13 marks

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X4 A drone flies 120 km along a straight route and then straight back. Its speed in still air is v km/h, where $v > 10$. A wind blows at 10 km/h along the route, against the drone on the way out and with it on the way back. Model the speed over the ground as $v - 10$ on the way out and $v + 10$ on the way back.

(a) Show that the time for the round trip, t hours, is given by $t = \dfrac{240v}{v^2 - 100}$. [3 marks]

(b) The round trip takes 5 hours. Find v. [3 marks]

Answer $v = \ldots$

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(c) The drone's battery allows at most 4 hours of flying. Find, in the form $p + q\sqrt{10}$, the least still-air speed for which the drone can complete the round trip. [4 marks]

Answer Least speed $=$ ... km/h

(d) Prove that, with a wind of any speed w km/h, where $0 < w < v$, the round trip takes longer than it would with no wind. State one way in which the model may not fit a real drone. [3 marks]

Total for X4: 13 marks

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X5 [9 marks]

$$f(x) = x^2 - 2ax + 3, \qquad 0 \leq x \leq 4,$$

where a is a constant. The least value of $f(x)$ is -6. Find the value of a. You must consider every possible position of the vertex of the graph.

Answer $a = \ldots$

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X6 The line l has equation $y = kx$, where k is a constant. The curve C has equation $y = x^2 - 2x + 4$.

(a) Find the set of values of k for which l meets C at two distinct points. [4 marks]

Answer ...

(b) For each value of k for which l is a tangent to C, find the point of contact. [2 marks]

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(c) When l meets C at two distinct points A and B, let M be the midpoint of AB. Show that, as k varies, M lies on the curve $y = 2x^2 - 2x$, and state the set of possible values of the x-coordinate of M. [6 marks]

Answer x -coordinate of M : ...

Total for X6: 12 marks

X7 k is a constant and

$$x^4 - 8x^2 + k = 0.$$

(a) By using the substitution $u = x^2$, show that $u = 4 \pm \sqrt{16 - k}$ when $k \leq 16$. [2 marks]

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(b) Find the set of values of k for which the equation has four distinct real roots. [3 marks]

Answer ...

(c) Solve the equation when $k = 0$, and explain why it has exactly three distinct real roots. [2 marks]

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(d) Find the set of values of k for which the equation has exactly two distinct real roots. [4 marks]

Answer ...

Total for X7: 11 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use the discriminant to find the range of a function or the values of a constant.	○	○	○
I can use the sum and product of the roots without solving the quadratic.	○	○	○
I can split a question into cases and reject answers that do not fit the case.	○	○	○
I can turn a model into a quadratic inequality and criticise the model.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Multiply up: $y(x^2 + 2) = 2x + 1$. Collect everything on one side as a quadratic in x whose coefficients involve y.

Hint 2 A value y is in the range only if this quadratic has a real solution x. Write down the condition on its discriminant.

Hint 3 The discriminant condition is a quadratic inequality in y. At each end of the range the discriminant is 0, so the quadratic in x has a repeated root: that root is where f is greatest or least.

X2

Hint 1 A line is a tangent to a parabola when the equation you get by putting them equal has a repeated root.

Hint 2 For (b), write each tangent as $y = mx - \dfrac{m^2}{4}$ and solve the two equations together. A difference of two squares appears.

Hint 3 For (d) you know the sum and the product of the two gradients, so they are the roots of one quadratic.

X3

Hint 1 Write $a + b$ in terms of c and ab in terms of the area. Then $(a + b)^2 = a^2 + b^2 + 2ab = c^2 + 2ab$.

Hint 2 Once you know $a + b$ and ab, the sides a and b are the roots of $t^2 - (a + b)t + ab = 0$. They exist only if that quadratic has real roots.

Hint 3 For (c), real roots needs $(a + b)^2 \geq 4ab$. This gives a quadratic inequality in A. Use $c > 0$ to decide which part of its solution is possible.

X4

Hint 1 Time $=$ distance $\div$ speed. Write the time for each leg, then add the two fractions.

Hint 2 For (b) and (c), multiply through by $v^2 - 100$. Because $v > 10$ this is positive, so an inequality keeps its direction.

Hint 3 For (d), redo (a) with w in place of 10, then compare the denominator $v^2 - w^2$ with v^2.

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X5

Hint 1 Complete the square to find the vertex of $y = f(x)$ in terms of a.

Hint 2 The vertex may be inside the interval $0 \leq x \leq 4$ or outside it. Sketch each case: where is the least value on the interval?

Hint 3 In each case form an equation for a, solve it, and check the answer really is in that case.

X6

Hint 1 Put $kx = x^2 - 2x + 4$ and collect the terms into a quadratic in x.

Hint 2 The line is a tangent when the discriminant is 0. For (c), you do not need to solve the quadratic: the sum of its roots is minus the coefficient of x.

Hint 3 The midpoint's x-coordinate is half the sum of the roots, and M lies on the line $y = kx$. Write k in terms of X and substitute. Use (a) to find the possible X.

X7

Hint 1 With $u = x^2$ the equation is a quadratic in u. Each value of u gives the values $x = \pm\sqrt{u}$.

Hint 2 Count the roots: a positive u gives two values of x, $u = 0$ gives one, and a negative u gives none.

Hint 3 The product of the two values of u is k and their sum is 8. Use the signs of these to decide when the values of u are positive, negative or zero.

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