A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 3: EQUATIONS AND INEQUALITIES · A* EXAM PRACTICE

Hints online, answers free with an account:
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Equations and inequalities: A* questions

71 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 A circle C has equation $x^2 + y^2 - 10x - 4y + 20 = 0$. The line l has equation $y = kx$, where k is a constant. Given that l meets C at two distinct points, find the set of possible values of k. Give your answer in set notation, with the limits in exact form. [8 marks]

Answer $\{k : \ldots\}$

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X2 The line $2x + y = k$, where k is a constant, and the curve $x^2 + xy + y^2 = 12$ are drawn on the same axes.

(a) Show that the x-coordinates of any points where the line meets the curve satisfy $3x^2 - 3kx + k^2 - 12 = 0$. [3 marks]

(b) Find the set of values of k for which the line and the curve have at least one point in common. Give the limits in surd form. [4 marks]

Answer $\{k : \ldots\}$

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(c) Hence find the greatest value of $2x + y$ for points (x, y) on the curve, and the coordinates of the point where it occurs. [3 marks]

Answer greatest value $= \ldots$ at $(\ldots, \ldots)$

Total for X2: 10 marks

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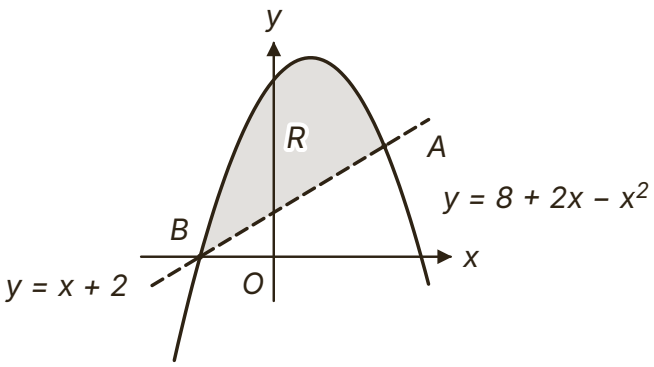
X3 The diagram shows the curve with equation $y = 8 + 2x - x^2$ and the line with equation $y = x + 2$. The line is drawn dashed. The line meets the curve at A and B. The shaded region R, between the line and the curve, does not include the points on the dashed line. It includes the points on the curve between A and B, but not A and B themselves, which lie on the dashed line.



(a) Write down the two inequalities that define R. [2 marks]

Answer ... and ...

(b) Find the coordinates of A and B. [3 marks]

Answer $A(\ldots, \ldots)$, $B(\ldots, \ldots)$

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(c) The point (x, y) lies in R. Find the greatest possible value of $2x + y$, showing that the point where it occurs lies in R. [5 marks]

Answer greatest value $= \ldots$

(d) Explain why $2x + y$ has no least value for points in R. [2 marks]

Total for X3: 12 marks

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X4 In this question m and c are constants. The line $y = mx + c$ is not vertical.

(a) Show that the line $y = mx + c$ is a tangent to the curve $y = x^2$ if and only if $m^2 + 4c = 0$. [3 marks]

(b) The point $P(p, -\frac{1}{4})$ is fixed. Show that the gradients m of the tangents to $y = x^2$ that pass through P satisfy $m^2 - 4pm - 1 = 0$. [3 marks]

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(c) Prove that, for every value of p, there are exactly two tangents to $y = x^2$ through P, and that these two tangents are perpendicular. [3 marks]

Total for X4: 9 marks

X5 Find the set of values of x for which both $x^3 - 3x^2 - 10x + 24 \leqslant 0$ and $2x^2 - 3x - 9 > 0$. Give your answer in set notation. [8 marks]

Answer ...

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X6 The curve C_1 has equation $y = x^2 - 4$ and the circle C_2 has equation $x^2 + y^2 = r^2$, where r is a positive constant.

(a) Show that the y-coordinates of any points where C_1 and C_2 meet satisfy $y^2 + y + 4 - r^2 = 0$. [2 marks]

(b) Given that $r = 4$, find the coordinates of all the points where C_1 and C_2 meet. Explain why there are three points and not four. [4 marks]

Answer ...

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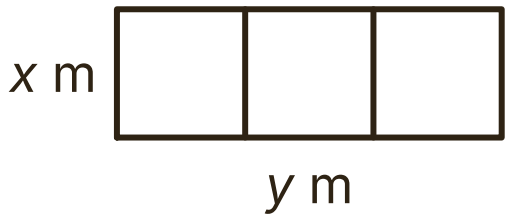
(c) Find the set of values of r for which C_1 and C_2 meet at exactly four points. Give the limits in exact form. [5 marks]

Answer $\ldots < r < \ldots$

Total for X6: 11 marks

X7 A farmer has 120 m of fencing to make a rectangular enclosure divided into three equal rectangular pens, as shown. The enclosure is y m long and x m wide. All 120 m of fencing is used for the outside and the two dividers.

(a) Show that $y = 60 - 2x$. Given that the area of each pen is at least $120\ \text{m}^2$, show that $x^2 - 30x + 180 \leqslant 0$. [3 marks]



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(b) Each pen must also be at least as long, along the length of the enclosure, as it is wide. Find the set of possible values of x. Give the limits in exact form. [5 marks]

Answer $\{x : \ldots\}$

(c) Ravi says: "The total area is $x(60 - 2x)$, which is greatest when $x = 15$, so the largest enclosure has area $450\ \mathrm{m}^2$." Explain why Ravi is wrong, and find the largest possible total area. [3 marks]

Answer largest total area $= \ldots$

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(d) Give two reasons why the model might not give the real largest area. [2 marks]

Total for X7: 13 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use the discriminant to find the values of a constant for which a line and a curve meet, touch or miss.	○	○	○
I can solve simultaneous equations where one is a curve or a circle, and count the points of intersection.	○	○	○
I can solve quadratic and cubic inequalities and give the answer in set notation.	○	○	○
I can describe a region with inequalities and find the greatest value of an expression in it.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Put $y = kx$ into the circle's equation and collect the terms into a quadratic in x.

Hint 2 Two distinct points means two distinct real roots of that quadratic: $b^2 - 4ac > 0$. This gives a quadratic inequality in k.

Hint 3 Solve $16k^2 - 20k - 5 = 0$ with the formula. The inequality $16k^2 - 20k - 5 < 0$ holds between the two roots.

X2

Hint 1 Make y the subject of the line, $y = k - 2x$, and substitute into the curve.

Hint 2 "At least one point in common" includes the line touching the curve, so use $b^2 - 4ac \geqslant 0$.

Hint 3 Every point on the curve lies on some line $2x + y = k$. The largest k that still meets the curve is the greatest value. At that k the root is repeated, so $x = -\dfrac{b}{2a}$.

X3

Hint 1 The curve is solid, so its inequality uses $\leqslant$. The line is dashed, so its inequality is strict.

Hint 2 $2x + y = c$ is a family of parallel lines with gradient -2. The largest c that still meets R touches the curve. Put $y = c - 2x$ into the curve and ask when the quadratic in x has a repeated root.

Hint 3 For the least value, look at the corners of R. Is the corner with the smallest $2x + y$ part of R?

X4

Hint 1 A line is a tangent to a parabola when the quadratic you get from solving them together has a repeated root.

Hint 2 P lies on the line, so $-\dfrac{1}{4} = mp + c$. Use this to remove c from $m^2 + 4c = 0$.

Hint 3 For $am^2 + bm + c = 0$ the product of the roots is $\dfrac{c}{a}$. Two lines with gradients whose product is -1 are perpendicular.

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X5

Hint 1 Try small whole numbers in the cubic to find a root, then use the factor theorem and divide.

Hint 2 Sketch the cubic through its three roots. A positive cubic is below the axis to the left of its smallest root and between its middle and largest roots.

Hint 3 Draw both solution sets on one number line and keep only the parts in both. Watch which ends are included.

X6

Hint 1 From $y = x^2 - 4$ you can write $x^2 = y + 4$. Substitute x^2, not x.

Hint 2 Each value of y gives points with $x^2 = y + 4$. Count the solutions of that for each y.

Hint 3 For four points you need two different values of y, both bigger than -4. Use the discriminant for the first, and solve for the smaller root for the second. Part (b) shows what happens at $r = 4$.

X7

Hint 1 Count the fencing: two lengths of y and four widths of x (two outside edges and two dividers).

Hint 2 Each pen is $\frac{y}{3}$ long and x wide. Write "at least as long as it is wide" as an inequality in x, then combine it with the first one on a number line.

Hint 3 The total area is a quadratic in x with its maximum at $x = 15$. On an interval that stops before 15, where is it largest?

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