Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-year-1-chapter-4-a-star/

Graphs and transformations: A* questions

73 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The curve C_1 has equation $y = x(x-2)(x+2)$. The curve C_2 has equation $y = \dfrac{3}{x}$, $x \neq 0$.

(a) On the same axes, sketch C_1 and C_2. Show the coordinates of the points where C_1 meets the axes, and the asymptotes of C_2. State the number of points where C_1 and C_2 meet. [4 marks]

(b) Find the exact x-coordinates of the points where C_1 and C_2 meet. [4 marks]

Answer $x = \ldots$

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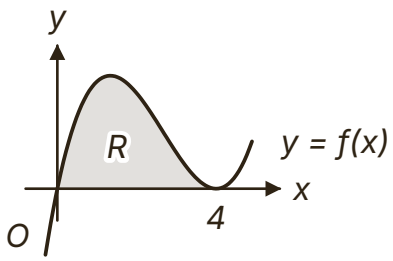
(c) The curve C_3 has equation $y = \dfrac{k}{x}$, where k is a non-zero constant. Find the set of values of k for which C_1 and C_3 meet at exactly four points. [5 marks]

Answer $\{k : \ldots\}$

Total for X1: 13 marks

X2 The diagram shows part of the curve with equation $y = f(x)$, where $f(x) = x(x - 4)^2$. The region R lies between the curve and the x-axis for $0 \leqslant x \leqslant 4$.

(a) Find the area of R. [4 marks]



Answer area $= \ldots$

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(b) Hence find the area between the curve with equation $y = 3f\left(\frac{x}{2}\right)$ and the x-axis for $0 \leqslant x \leqslant 8$. [3 marks]

Answer area $= \ldots$

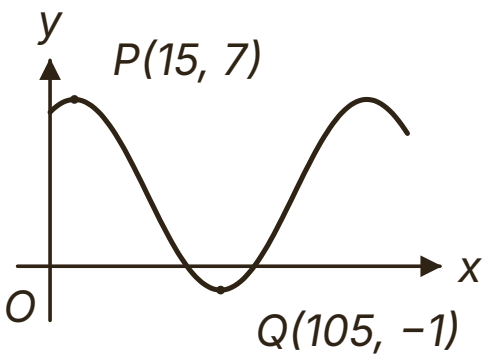
(c) Find the area of the region bounded by the curve with equation $y = f(2x) + 2$, the coordinate axes and the line $x = 2$. [3 marks]

Answer area $= \ldots$

Total for X2: 10 marks

X3 In this question x is in degrees. The curve shown has equation $y = a\sin(bx + c) + d$, where a, b, c and d are constants with $a > 0$, $b > 0$ and $0 < c < 180$. The curve has a maximum point $P(15, 7)$. The next minimum point after P is $Q(105, -1)$.

(a) Find the values of a and d. [2 marks]



Answer $a = \ldots$, $d = \ldots$

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(b) Find the values of b and c. [4 marks]

Answer $b = \ldots$, $c = \ldots$

(c) Describe fully a sequence of transformations that maps the curve $y = \sin x$ onto this curve. [3 marks]

(d) Solve $a\sin(bx + c) + d = 6$ for $0 \leqslant x \leqslant 360$. Give your answers to 1 decimal place. [3 marks]

Answer $x = \ldots$

Total for X3: 12 marks

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X4 The curve C has equation $y = x^2 - 4x + 9$. The curve C is stretched parallel to the x-axis with scale factor k, where $k > 0$, and then translated by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$. The image of C touches the line $y = x$. Find the exact value of k. [8 marks]

Answer $k = \ldots$

X5 A team of n workers paints a sports hall. A model for the time taken, T hours, is $T = \frac{a}{n} + b$, where a and b are positive constants. With 4 workers the job takes 17 hours. With 10 workers it takes 8 hours.

(a) Find the values of a and b. [3 marks]

Answer $a = \ldots$, $b = \ldots$

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(b) Describe two transformations that map the graph of $y = \frac{1}{x}$ onto the graph of $T = \frac{a}{n} + b$, and explain what the horizontal asymptote means in this context. [3 marks]

(c) Find the least number of workers needed for the job to take less than 5 hours. [3 marks]

Answer ... workers

(d) Give two reasons why the model may not be reliable for large values of n. [2 marks]

Total for X5: 11 marks

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X6 The curve C_1 has equation $y = 2^x$. The curve C_1 is translated by $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ and the result is then stretched parallel to the y-axis with scale factor 3, giving the curve C_2. Find the equation of C_2 and the equation of its asymptote. Find the exact x-coordinate of the point where C_2 crosses the x-axis, and the exact coordinates of the point where C_1 and C_2 meet. [9 marks]

Answer C_2 : $y = \ldots$; asymptote $\ldots$; $x = \ldots$; $(\ldots, \ldots)$

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X7 $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 9$

(a) Show that $f(x) = (x^2 - 4x + 3)^2$. Hence sketch the curve $y = f(x)$, showing the coordinates of the points where it meets the axes. [4 marks]

(b) Show that $f(x + 2) = (x^2 - 1)^2$. Hence explain why the curve $y = f(x)$ is symmetric about the line $x = 2$. [3 marks]

(c) Solve $f(x) = 4$. Give your answers in exact form. [3 marks]

Answer $x = \ldots$

Total for X7: 10 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can sketch cubic, quartic and reciprocal graphs and use them to count and find points of intersection.	○	○	○
I can find the values of a constant for which two graphs meet a given number of times.	○	○	○
I can describe and apply translations and stretches, in the right order, to any graph.	○	○	○
I can use transformations to find equations, asymptotes, symmetries and areas.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 C_1 crosses the x-axis at -2, 0 and 2. C_2 lies in the first and third quadrants. Where can a branch of C_2 meet the cubic?

Hint 2 Multiply $x^3 - 4x = \dfrac{3}{x}$ by x. The quartic has only even powers, so put $u = x^2$. A value of u is only useful if it is positive.

Hint 3 Each positive root u gives two values of x. For four points the quadratic in u needs two distinct roots that are both positive: use the discriminant, and the sum and product of the roots.

X2

Hint 1 Expand $x(x-4)^2$ before integrating. The curve touches the x-axis at $x = 4$, so the region is above the axis all the way.

Hint 2 A stretch parallel to the x-axis by scale factor 2 doubles every width, so it doubles the area. The new limits 0 to 8 match the old limits 0 to 4.

Hint 3 $f(2x)$ over $0 \leqslant x \leqslant 2$ squeezes R to half its width. The $+2$ lifts the whole curve, so a rectangle of height 2 is added underneath.

X3

Hint 1 The maximum value is $a + d$ and the minimum value is $d - a$.

Hint 2 A maximum and the next minimum are half a period apart. Then put $x = 15$ into $bx + c$: at a maximum of a sine curve the angle is 90 (plus a multiple of 360).

Hint 3 Change the angle first: $\sin(x + 60)$, then $\sin(2x + 60)$. Then change the output: multiply by 4, then add 3. For (d), find the interval that $2x + 60$ lies in before solving.

X4

Hint 1 A stretch parallel to the x-axis with scale factor k replaces x by $\dfrac{x}{k}$. A translation by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ subtracts 3.

Hint 2 "Touches the line $y = x$" means the equation image $= x$ has a repeated root.

Hint 3 Multiply the discriminant condition by k^2 to clear the fractions. You should get a perfect square equal to 24. Remember $k > 0$.

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X5

Hint 1 Put each pair of values into the model to get two simultaneous equations in a and b.

Hint 2 $\dfrac{60}{n}$ is $\dfrac{1}{n}$ stretched vertically, and the $+2$ moves the graph up. As n gets very large, $\dfrac{60}{n}$ gets close to 0.

Hint 3 Solve $\dfrac{60}{n} + 2 < 5$. Because n is positive you can multiply by n without reversing the sign. Then the answer must be a whole number of workers.

X6

Hint 1 A translation by $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ replaces x by $x - 1$ and subtracts 6. A stretch parallel to the y-axis multiplies the whole right-hand side, including the -6.

Hint 2 The asymptote of $y = 2^x$ is $y = 0$. Follow what happens to that line under the two transformations.

Hint 3 To solve $2^x = 3 \times 2^{x-1} - 18$, write $2^{x-1} = \dfrac{1}{2} \times 2^x$ so that there is only one power of 2.

X7

Hint 1 Expand $(x^2 - 4x + 3)(x^2 - 4x + 3)$ term by term. Then factorise $x^2 - 4x + 3$: a square of a product touches the axis at each root.

Hint 2 Replace x by $x + 2$ inside the bracket only. A function with only even powers of x has a graph symmetric about the y-axis.

Hint 3 If $g^2 = 4$ then $g = 2$ or $g = -2$. Solve both quadratics and check each one has real roots.

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