A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 6: CIRCLES · A* EXAM PRACTICE

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Circles: A* questions

79 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 A circle lies in the first quadrant and touches the positive x-axis, the positive y-axis and the line $3x + 4y = 24$. Find the equations of the two circles that do this, and the coordinates of the point where each one touches the line $3x + 4y = 24$. [9 marks]

Answer $(x - \ldots)^2 + (y - \ldots)^2 = \ldots$ and $(x - \ldots)^2 + (y - \ldots)^2 = \ldots$

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X2 For each real number k, the circle C_k has equation $(x-k)^2+(y-2k)^2=20$.

(a) Show that the centres of all the circles C_k lie on one straight line, and give its equation. [2 marks]

Answer $y=\ldots$

(b) Find the set of values of k for which C_k meets the line $y=x$ at two distinct points. [4 marks]

Answer $\cdots<k<\ldots$

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(c) For some values of k, the line $y = x$ cuts a chord of length 6 from C_k. Find these values of k, giving them in exact form. [4 marks]

Answer $k = \ldots$

(d) Find the values of k for which C_k touches the x-axis. [2 marks]

Answer $k = \ldots$

Total for X2: 12 marks

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X3 The circle C has centre $(0, 3)$ and radius r. The curve P has equation $y = x^2$.

(a) Show that the y-coordinates of the points where C meets P satisfy $y^2 - 5y + 9 - r^2 = 0$. [2 marks]

(b) The circle touches the curve at two points. Find the exact value of r and the exact coordinates of these two points. [4 marks]

Answer $r = \ldots$, points $\ldots$

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(c) Find the set of values of r for which C meets P at exactly four points. [4 marks]

Answer $\cdots < r < \ldots$

(d) State the number of points where C meets P when $r = 3$, and when $r = 4$, giving a reason in each case. [2 marks]

Total for X3: 12 marks

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X4 The circle C_1 has equation $x^2 + y^2 = 9$. The circle C_2 has centre $B(k, 0)$, where $k > 0$, and radius 4. The circles meet at P and Q, where P is above the x-axis. The tangent to C_1 at P passes through B. Find k and the coordinates of P and Q. Show that the tangent to C_2 at P passes through the origin O, and find the length of PQ. [9 marks]

Answer $k = \ldots$, $PQ = \ldots$

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X5 The circle C has equation $x^2 + y^2 - 4x - 6y - 12 = 0$. The point P is $(4, 4)$.

(a) Find the centre A and the radius of C, and show that P lies inside C. [3 marks]

(b) A chord of C passes through P and has midpoint M. Explain why the chord has length $2\sqrt{25 - AM^2}$ and why $AM \leq \sqrt{5}$. Hence find the equation of the shortest chord through P, and its exact length. [5 marks]

Answer $y = \ldots$, length $\ldots$

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(c) Explain why no chord of C through P has length 8. [2 marks]

(d) Find the equations of the two chords of C through P that have length $4\sqrt{6}$. [4 marks]

Answer ... and ...

Total for X5: 14 marks

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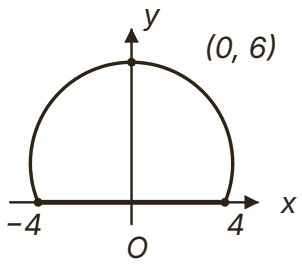
X6 The entrance to a road tunnel is modelled as an arc of a circle standing on the x-axis, as in the figure. The road is the x-axis from $(-4, 0)$ to $(4, 0)$, and the highest point of the tunnel is $(0, 6)$. Units are metres.

(a) Show that the circle has centre $\left(0, \frac{5}{3}\right)$ and radius $\frac{13}{3}$. [4 marks]



(b) A lorry is 3 m wide and 5 m tall. It drives along the middle of the road. Show that it can pass through the tunnel. [3 marks]

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(c) Find the greatest width of a lorry 5 m tall that can pass through the tunnel when it drives along the middle of the road. Give your answer in metres to 3 significant figures. [3 marks]

Answer ... m

(d) Give one reason why the tunnel's operators would not allow a lorry this wide, 5 m tall, through the tunnel. [2 marks]

Total for X6: 12 marks

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X7 The circle C has equation $x^2 + y^2 = 1$, and A is the point $(-1, 0)$. For each real number t, the line through A with gradient t meets C again at the point P_t.

(a) Show that P_t has coordinates $\left(\dfrac{1 - t^2}{1 + t^2}, \dfrac{2t}{1 + t^2}\right)$. [4 marks]

(b) Find the value of t for which P_t is $\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$, and the value of t for which P_t is $\left(\dfrac{8}{17}, -\dfrac{15}{17}\right)$. [3 marks]

Answer $t = \ldots$ and $t = \ldots$

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(c) Use $t = \frac{2}{7}$ to find positive integers a and b with $a^2 + b^2 = 53^2$ [4 marks]

. Show that your answer is correct.

Answer $a = \ldots$, $b = \ldots$

Total for X7: 11 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use the discriminant to decide how a circle meets a line or a curve.	○	○	○
I can use the fact that a tangent is perpendicular to the radius.	○	○	○
I can use the perpendicular from the centre to a chord.	○	○	○
I can model a real shape with a circle and say where the model stops working.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 If a circle touches the x-axis, its radius equals the y-coordinate of its centre. It also touches the y-axis, so its centre is (a, a).

Hint 2 Substitute the line into the circle. Touching means the quadratic in x has a repeated root.

Hint 3 The condition gives a quadratic in a with two positive roots. One circle is inside the triangle made by the line and the axes, and the other is outside it.

X2

Hint 1 Read the centre of C_k from its equation and see what its coordinates have in common.

Hint 2 Substitute $y = x$. Two distinct points means the discriminant is positive.

Hint 3 For the chord, the two meeting points have x-coordinates x_1 and x_2, and a step of d along $y = x$ has length $\sqrt{2}\,d$. You can find $(x_1 - x_2)^2$ from the discriminant.

X3

Hint 1 Write the equation of the circle and replace x^2 by y.

Hint 2 Every positive value of y gives two points on $y = x^2$, one each side of the y-axis. So the circle touches the curve twice when the quadratic in y has one repeated positive root.

Hint 3 For four points the quadratic in y needs two different positive roots. Use the discriminant, then the sum and product of the roots.

X4

Hint 1 A tangent is perpendicular to the radius at the point where it touches. Which angle does that make $90°$?

Hint 2 Triangle OPB has a right angle at P, with $OP = 3$ and $BP = 4$.

Hint 3 Subtract one circle equation from the other to find the x-coordinate of P and Q. For the tangent to C_2, compare the gradients of BP and OP.

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X5	**Hint 1**	Complete the square to find the centre and radius. A point is inside the circle when its distance from the centre is less than the radius.
	Hint 2	The line from the centre to the midpoint M of a chord is perpendicular to the chord. Use Pythagoras in the triangle with the centre, M and one end of the chord, and in triangle AMP.
	Hint 3	For a chord of length $4\sqrt{6}$, find AM and then MP. M is on a circle about A and on a circle about P.
X6	**Hint 1**	By symmetry the centre is on the y-axis. It is the same distance from $(0, 6)$ as from $(4, 0)$.
	Hint 2	The lorry first touches the tunnel at its top corners. Find the height of the tunnel at $x = 1.5$.
	Hint 3	For the widest lorry, put $y = 5$ into the equation of the circle and solve for x.
X7	**Hint 1**	The line through A with gradient t is $y = t(x + 1)$. Substitute it into the circle.
	Hint 2	You know one root of the quadratic already: $x = -1$, because A is on both. Use the product of the roots, or factorise.
	Hint 3	t is the gradient of the line from A to P_t. For (c), a point on $x^2 + y^2 = 1$ with denominators 53 gives integers whose squares add to 53^2.

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