A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 7: ALGEBRAIC METHODS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Algebraic methods: A* questions

76 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 $p(x) = 2x^3 + ax^2 + bx + c$, where a, b and c are constants. $(x - 1)$ is a factor of $p(x)$. When $p(x)$ is divided by $(x + 1)$ the remainder is -2. The curve $y = p(x)$ has gradient 15 at the point where $x = 1$.

(a) Find the values of a, b and c. [6 marks]

Answer $a = \ldots$, $b = \ldots$, $c = \ldots$

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(b) Hence factorise $p(x)$ completely. [3 marks]

Answer .....

(c) The curve $y = p(x)$ and the x-axis enclose two finite regions. Find the total area of the two regions. Give your answer as a fraction in its simplest form. [6 marks]

Answer total area $= \ldots$

Total for X1: 15 marks

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X2 Prove that the sum of the cubes of any three consecutive integers is a multiple of 9. [8 marks]

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X3 The curve C has equation $y = \dfrac{2x^2 + 5x - 3}{x^2 + 2x - 3}$, $x \neq 1$, $x \neq -3$. The point $A(0, 1)$ lies on C.

(a) Show that the equation of C can be written as $y = 2 + \dfrac{1}{x - 1}$ [3 marks]

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(b) The line l passes through A and has gradient m, where $m \neq 0$. Show that the x-coordinates of the points where l meets C satisfy $x(mx - m - 1) = 0$. [3 marks]

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(c) Find the value of m for which l is a tangent to C at A. [2 marks]

Answer $m = \ldots$

(d) Show that there is another value of m, $m \neq 0$, for which l meets C at A only, and find it. Explain your answer. [3 marks]

Total for X3: 11 marks

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X4 Solve, for $0 \leq \theta < 360°$, [9 marks]

$$4\cos^3\theta - \cos\theta + 3 = 4\sin^2\theta.$$

Show each stage of your working.

Answer $\theta = \ldots$

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X5 In this question p is a prime number with $p > 3$.

(a) By writing every integer in one of the forms $6k$, $6k + 1$, $6k + 2$, $6k + 3$, $6k + 4$ or $6k + 5$, prove that p is of the form $6k + 1$ or $6k + 5$. [4 marks]

(b) Hence prove that $p^2 - 1$ is a multiple of 24. [5 marks]

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(c) Ali says: "If $n^2 - 1$ is a multiple of 24, then n must be a prime number." Show that Ali is wrong. [2 marks]

Total for X5: 11 marks

X6 Water is pumped into and out of a tank. The volume of water in the tank, V litres, t hours after midday, is modelled by

$$V = t^3 - 12t^2 + 36t + 20, \quad 0 \leq t \leq 8.$$

(a) Find the volume of water at 2 pm, and show that $V - 52 = (t-2)^2(t-8)$. Hence find the other time when the volume is the same as at 2 pm. [4 marks]

Answer $V = \ldots$ litres; other time $\ldots$

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(b) Hence, without differentiating, show that the model never gives more than 52 litres in the tank. [3 marks]

(c) Use calculus to find the least volume of water in the tank, and the times when it occurs. [4 marks]

Answer least volume ... litres at ...

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(d) Explain why this model should not be used for $t > 8$. Support your answer with a calculation. [2 marks]

Total for X6: 13 marks

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X7 Prove that, for every integer n, the product of four consecutive integers $n(n+1)(n+2)(n+3)$ is one less than a square number. Hence find four consecutive positive integers whose product is 3024. [9 marks]

Answer the integers are ...

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use the factor theorem and the remainder to find unknown constants.	○	○	○
I can simplify an algebraic fraction and use the values it leaves out.	○	○	○
I can prove a result by exhaustion, splitting into the cases that cover every integer.	○	○	○
I can show a statement is false with a counter-example.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Each fact gives one equation: $p(1) = 0$, $p(-1) = -2$ and $p'(1) = 15$.

Hint 2 Divide $p(x)$ by $(x - 1)$ and factorise the quadratic you get.

Hint 3 Between which roots is the curve above the axis, and between which is it below? Find each area with its own integral and add their sizes.

X2

Hint 1 Call the middle integer m. The other two are $m - 1$ and $m + 1$, and the working is shorter.

Hint 2 The sum simplifies to $3m(m^2 + 2)$. You still need one more factor of 3.

Hint 3 Every integer is $3k$, $3k + 1$ or $3k + 2$. Check that $m(m^2 + 2)$ is a multiple of 3 in each case.

X3

Hint 1 Factorise $2x^2 + 5x - 3$ and $x^2 + 2x - 3$. They share a factor.

Hint 2 Use the simpler form $\dfrac{2x - 1}{x - 1}$ and multiply both sides by $x - 1$.

Hint 3 The line meets C at one point if the two roots are equal, or if the second root is a value of x that C leaves out.

X4

Hint 1 Write everything in terms of $\cos\theta$ with $\sin^2\theta = 1 - \cos^2\theta$.

Hint 2 You get a cubic in $c = \cos\theta$. Try $c = 1$ and $c = -1$ in the factor theorem.

Hint 3 Each value of $\cos\theta$ between -1 and 1 gives two angles in the interval, except $\cos\theta = -1$, which gives one.

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X5

Hint 1 Which of the six forms are always even? Which is always a multiple of 3?

Hint 2 $6k + 5 = 6(k + 1) - 1$, so every such prime is $6j + 1$ or $6j - 1$. Square and subtract 1, then think about whether j is even or odd.

Hint 3 Look for n of the form $6j \pm 1$ that is not prime. The smallest such n is quite small.

X6

Hint 1 Put $t = 2$ into V, then expand $(t - 2)^2(t - 8)$ and compare with $V - 52$.

Hint 2 A square is never negative. What sign is $t - 8$ for these values of t?

Hint 3 Find where $\dfrac{dV}{dt} = 0$, and remember to check the end points $t = 0$ and $t = 8$ as well.

X7

Hint 1 Multiply the outside pair $n(n + 3)$ and the inside pair $(n + 1)(n + 2)$ separately.

Hint 2 The two products differ by 2. Call the number half way between them m.

Hint 3 $3024 + 1 = 3025$. Which number squared is 3025? Set it equal to $n^2 + 3n + 1$.

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