A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 1 · CHAPTER 8: THE BINOMIAL EXPANSION · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-year-1-chapter-8-a-star/

The binomial expansion: A* questions

79 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 θ is an angle with $0 \leq \theta < 360°$, $\theta \neq 90°$, $\theta \neq 270°$. In the binomial expansion of $(2 + x\tan\theta)^5$, in ascending powers of x,

(a) find the coefficients of x^2 and of x^3, in terms of $\tan\theta$. [3 marks]

Answer x^2 : ... x^3 : ...

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(b) The coefficients of x^2 and x^3 are equal. Find all the possible values of θ. Give your answers to 1 decimal place where necessary. [6 marks]

Answer $\theta = \ldots$

(c) Instead, the sum of all the coefficients in the expansion is 1. Find the possible values of θ. [3 marks]

Answer $\theta = \ldots$

Total for X1: 12 marks

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X2 n is a positive integer. In the binomial expansion of $(1+x)^n$ there are three consecutive terms whose coefficients are in the ratio $1:2:3$. Find the value of n and the three coefficients. [8 marks]

X3 $f(x) = (1+x)^6 + (1-x)^6$.

(a) Show that $f(x) = 2 + 30x^2 + 30x^4 + 2x^6$. [3 marks]

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(b) Hence explain why $f(x) \geq 2$ for all real x, and state when $f(x) = 2$. [2 marks]

(c) Use calculus to show that the curve $y = f(x)$ has exactly one stationary point. Find it and determine its nature. [4 marks]

Answer stationary point $(\ldots, \ldots)$, a ...

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(d) Find the exact value of $\int_{-1}^{1} f(x)\,dx$. [3 marks]

Answer ...

Total for X3: 12 marks

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X4 k is a constant. Find the set of values of k for which the coefficient of x and the coefficient of x^2 in the expansion of $(1+kx)^6(1-2x)^3$ are both negative. [10 marks]

Answer ...

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X5 In this question n is a positive integer.

(a) Prove that $r\binom{n}{r} = n\binom{n-1}{r-1}$ for $1 \leq r \leq n$. [3 marks]

(b) Show that $\binom{m}{0} + \binom{m}{1} + \binom{m}{2} + \cdots + \binom{m}{m} = 2^m$ for any positive integer m. [2 marks]

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(c) Hence show that $1\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} = n \times 2^{n-1}$. [4 marks]

(d) Given that $1\binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n} = 1024$, find n. Explain why there is only one value. [3 marks]

Answer $n = \ldots$

Total for X5: 12 marks

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X6 Money in a savings account earns 1% interest each month, added to the account at the end of the month. After n months, $£P$ grows to $£P \times 1.01^n$. Jo uses the approximation

$$1.01^n \approx 1 + \frac{n}{100} + \frac{n(n-1)}{20\,000}.$$

(a) Show how the binomial expansion gives this approximation. [3 marks]

(b) Use the approximation to find the least number of whole months for the money to increase by 20%. [4 marks]

Answer ... months

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(c) Use logarithms to find the least number of whole months exactly, and comment on the approximation. [3 marks]

Answer ... months

(d) Jo uses the approximation to find how long it takes for the money to double. Find the value she gets, compare it with the true value, and explain why her value is too large. [3 marks]

Answer approximation ... ; true ...

Total for X6: 13 marks

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X7 In the binomial expansion of $(1 + 3x)^n$, where n is a positive integer, the sum of the coefficients of x, x^2 and x^3 is 375.

(a) Show that $3n^3 - 6n^2 + 5n - 250 = 0$. [4 marks]

(b) Hence find n, showing that there is only one possible value. [4 marks]

Answer $n = \ldots$

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(c) Using this value of n, find the coefficient of x^2 in the expansion of $(1+3x)^n\left(3-\frac{1}{x}\right)^2$. [4 marks]

Answer ...

Total for X7: 12 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find unknowns from conditions on the coefficients of an expansion.	○	○	○
I can prove results about binomial coefficients with factorials and by putting x = 1.	○	○	○
I can use an expansion to estimate, and explain when an estimate is good or poor.	○	○	○
I can combine an expansion with other topics, such as inequalities and calculus.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 The term in x^r is $\binom{5}{r}2^{5-r}(x\tan\theta)^r$.

Hint 2 When you solve $80t^2 = 40t^3$, take out the common factor. Dividing by t^2 loses solutions.

Hint 3 The sum of all the coefficients is what the expansion equals when $x = 1$.

X2

Hint 1 Call the coefficients $\binom{n}{r}$, $\binom{n}{r+1}$ and $\binom{n}{r+2}$, and write each ratio of neighbours with factorials.

Hint 2 $\binom{n}{r+1} \div \binom{n}{r}$ simplifies to $\dfrac{n-r}{r+1}$. The second ratio is $\dfrac{3}{2}$, not 3.

Hint 3 You get two linear equations in n and r. Solve them, then check the three coefficients.

X3

Hint 1 Write out $(1+x)^6$ from Pascal's triangle. $(1-x)^6$ has the same coefficients with the signs alternating.

Hint 2 Every power of x left in $f(x)$ is even. What sign can an even power have?

Hint 3 Take out $12x$ from $f'(x)$. Can the bracket that is left ever be zero?

X4

Hint 1 You only need each expansion as far as the x^2 term.

Hint 2 The x^2 coefficient comes from three products: constant times x^2, x times x, and x^2 times constant.

Hint 3 Solve each inequality, then find the values of k that satisfy both.

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X5

Hint 1 Write both sides with factorials. $\dfrac{r}{r!} = \dfrac{1}{(r-1)!}$.

Hint 2 What does $(1+x)^m$ equal when $x = 1$?

Hint 3 Replace each $r\binom{n}{r}$ using part (a), take out the factor n, then use part (b).

X6

Hint 1 $1.01 = 1 + 0.01$. Use the first three terms of $(1+a)^n$ with $a = 0.01$.

Hint 2 Multiply the inequality by $20\,000$ to clear the fractions, and solve the quadratic.

Hint 3 Are the terms Jo ignores positive or negative? Is $n \times 0.01$ still small when n is about 70?

X7

Hint 1 The coefficient of x^r in $(1+3x)^n$ is $\binom{n}{r}3^r$. Write $\binom{n}{2}$ and $\binom{n}{3}$ in terms of n.

Hint 2 Try small whole numbers for n in the cubic. Once you have one root, divide to find a quadratic factor.

Hint 3 $\dfrac{6}{x}$ times a term in x^3 gives x^2. Which terms of $(1+3x)^5$ pair with each term of $9 - \dfrac{6}{x} + \dfrac{1}{x^2}$?

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