Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-year-1-chapter-9-a-star/

Trigonometric ratios: A* questions

81 marks

Date

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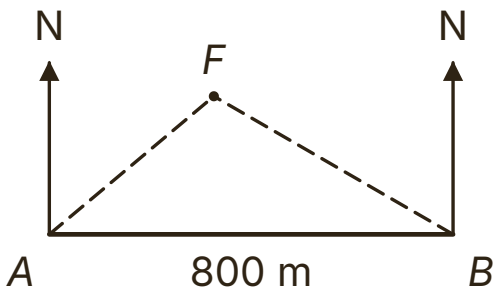
X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 A straight horizontal road runs due east from A to B, where $AB = 800$ m. A vertical tower stands on level ground with its foot at F. The bearing of F from A is $050°$ and the bearing of F from B is $300°$. From A, the angle of elevation of the top of the tower is $12°$. A cyclist rides along the road from A to B. Find the height of the tower and the greatest angle of elevation of the top of the tower seen by the cyclist. Give each answer to 3 significant figures. The figure is not to scale. [10 marks]



Answer height $= \ldots$ m; greatest angle of elevation $= \ldots$

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X2 In triangle ABC, $AB = 11$ cm, $BC = 7$ cm and angle $BAC = 32°$. Let $AC = x$ cm.

(a) Show that $x^2 - (22\cos 32°)x + 72 = 0$. [2 marks]

(b) Explain why there are two different triangles with these measurements. [3 marks]

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(c) The two triangles have areas Δ_1 and Δ_2, where $\Delta_1 > \Delta_2$. Without solving the equation in part (a), show that $\Delta_1 - \Delta_2 = \frac{11}{2}\sin 32^\circ\sqrt{484\cos^2 32^\circ - 288}$, and find its value to 3 significant figures. [4 marks]

Answer $\Delta_1 - \Delta_2 = \ldots$ cm²

Total for X2: 9 marks

X3 A triangle has sides of length x cm, $(x+1)$ cm and $(x+3)$ cm.

(a) Show that $x > 2$. [2 marks]

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(b) Find the set of values of x for which the triangle is obtuse. Give your answer in exact form. [4 marks]

Answer $\{x : \ldots\}$

(c) The cosine of the largest angle of the triangle is $-\frac{1}{4}$. Show that $3x^2 - 7x - 16 = 0$, and hence find the exact value of x. [4 marks]

Answer $x = \ldots$

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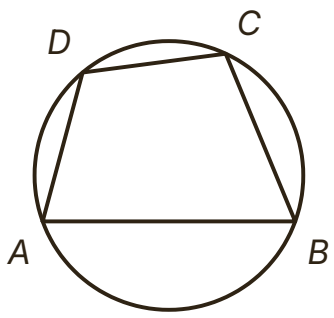
(d) For this value of x, find the area of the triangle, to 3 significant figures. [2 marks]

Answer area $=$ ... cm²

Total for X3: 12 marks

X4 The quadrilateral $ABCD$ is drawn inside a circle, with all four vertices on the circle. $AB = 8$ cm, $BC = 5$ cm, $CD = 3$ cm and $DA = 5$ cm. Find the exact area of $ABCD$ and the exact radius of the circle. The figure is not to scale. [8 marks]



Answer area $=$ ... cm²; radius $=$ ... cm

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X5 The circle C has equation $x^2 + y^2 = 25$ and centre O. The points $A(3, 4)$ and $B(4, -3)$ lie on C.

(a) Show that angle $AOB = 90°$. [2 marks]

(b) The point P lies on the major arc AB. Find angle APB, giving a reason. [2 marks]

Answer angle $APB = \ldots$

(c) Given that $PB = 5\sqrt{2}$, use the cosine rule in triangle APB to find AP. Hence find the coordinates of P. [4 marks]

Answer $AP = \ldots$; $P(\ldots, \ldots)$

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(d) Find the area of triangle APB. [2 marks]

Answer area $= \ldots$

Total for X5: 10 marks

X6 Relative to a fixed origin O, the points A, B and C have position vectors $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = 5\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OC} = 14\mathbf{i} + 4\mathbf{j}$.

(a) Find $|\overrightarrow{AB}|$, $|\overrightarrow{AC}|$ and $|\overrightarrow{BC}|$. [2 marks]

Answer $\ldots$, $\ldots$, $\ldots$

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(b) Show that $\cos(\angle BAC) = \dfrac{56}{65}$, and find the exact value of $\sin(\angle BAC)$. [4 marks]

Answer $\sin(\angle BAC) = \ldots$

(c) Find the exact area of triangle ABC. [2 marks]

Answer area $= \ldots$

(d) Find the exact shortest distance from A to the line BC. Give your answer in the form $\dfrac{p\sqrt{q}}{r}$. [2 marks]

Answer distance $= \ldots$

Total for X6: 10 marks

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X7 In triangle ABC, the line from A that bisects angle BAC meets BC at D. Write $a = BC$, $b = CA$ and $c = AB$.

(a) By using the sine rule in triangle ABD and in triangle ACD, prove that $\dfrac{BD}{DC} = \dfrac{c}{b}$. [4 marks]

(b) Given that $c = 6$ cm, $b = 10$ cm and $a = 12$ cm, find BD, and find the exact length of AD. [4 marks]

Answer $BD = \ldots$ cm; $AD = \ldots$ cm

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(c) Prove that, in any triangle, $AD^2 = bc\left(1 - \dfrac{a^2}{(b+c)^2}\right)$. [4 marks]

Total for X7: 12 marks

X8 In triangle ABC, $BC = a$, $CA = b$ and $AB = c$. The triangle is placed on coordinate axes with C at the origin and B at the point $(a, 0)$, with A above the x-axis.

(a) Explain why A has coordinates $(b\cos C, b\sin C)$. [2 marks]

(b) Hence prove the cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$. [3 marks]

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(c) Use the cosine rule to prove that $a = b\cos C + c\cos B$. [3 marks]

(d) Nina says: "Part (c) shows that, in every triangle, $a < b + c$." Is she right? Give a reason. [2 marks]

Total for X8: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can choose between the sine rule and the cosine rule, and use both in one problem.	○	○	○
I can explain how many triangles a set of measurements gives, and why.	○	○	○
I can use triangle results with circles, vectors and quadratic inequalities.	○	○	○
I can prove a triangle result from the sine rule or the cosine rule.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Find the three angles of triangle ABF from the bearings, then use the sine rule to find AF.

Hint 2 The tower's height is fixed, so the angle of elevation is largest when the cyclist is as close to F as possible.

Hint 3 The closest point of the road to F is where the perpendicular from F meets it. Check that this point is between A and B.

X2

Hint 1 Use the cosine rule with the 32° angle. The side opposite it is BC.

Hint 2 A quadratic has two different real roots when its discriminant is positive. The sum and the product of the roots tell you their signs.

Hint 3 Both triangles share the side AB and the angle at A, so each area is a fixed number times x. Find $(x_1 - x_2)^2$ from the sum and product of the roots.

X3

Hint 1 A triangle exists only if the two shorter sides are longer, together, than the longest side.

Hint 2 The obtuse angle, if there is one, faces the longest side. The cosine rule gives a negative cosine exactly when $c^2 > a^2 + b^2$.

Hint 3 $\sin C$ is positive for any angle of a triangle, so $\sin C = \sqrt{1 - \cos^2 C}$.

X4

Hint 1 The diagonal AC belongs to both triangle ABC and triangle ADC. Write AC^2 in two ways.

Hint 2 In a cyclic quadrilateral the opposite angles B and D add to 180°, so $\cos D = -\cos B$.

Hint 3 The circle is the circle through A, B and C. The angle at the centre subtended by AC is twice angle B.

X5

Hint 1 Find AB^2 with Pythagoras on the coordinates and compare it with $OA^2 + OB^2$.

Hint 2 Use a circle theorem: how is the angle at the circumference related to the angle at the centre?

Hint 3 After the cosine rule you get a quadratic in AP. A chord of length 10 in a circle of radius 5 is a diameter.

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X6

Hint 1 $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$. The magnitude of $a\mathbf{i} + b\mathbf{j}$ is $\sqrt{a^2 + b^2}$.

Hint 2 Use the cosine rule with the three lengths from part (a). $65^2 - 56^2$ factorises as a difference of two squares.

Hint 3 The shortest distance from A to BC is the height of the triangle when BC is the base.

X7

Hint 1 The angles ADB and ADC add to $180°$, so they have the same sine.

Hint 2 Split $BC = 12$ in the ratio $6 : 10$. Then find $\cos B$ from the whole triangle and use it again in triangle ABD.

Hint 3 Repeat part (b) with letters: $BD = \dfrac{ac}{b + c}$. Put the three terms over $(b + c)^2$, and look for the factor $(c - b)(c + b)$.

X8

Hint 1 Think of A as a point on a circle of radius b centred at C. Where is it, measured from the x-axis?

Hint 2 Find AB^2 from the two coordinates and simplify with $\sin^2 C + \cos^2 C = 1$.

Hint 3 Write $\cos C$ and $\cos B$ as fractions with the cosine rule, then add $b\cos C$ and $c\cos B$.

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