A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 1: ALGEBRAIC METHODS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Algebraic methods: A* questions

72 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 Prove by contradiction that the equation $x^3 + x + 1 = 0$ has no rational solutions. [9 marks]

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X2 In this question n, x and y are integers.

(a) Prove that n^2 leaves a remainder of 0 or 1 when it is divided by 4. [3 marks]

(b) Hence prove by contradiction that there are no integers x and y with $x^2 + y^2 = 2027$. [4 marks]

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(c) Sam says: "So every whole number that leaves a remainder of 1 when divided by 4 is the sum of two square numbers." Show that Sam is wrong. [3 marks]

Total for X2: 10 marks

X3 $f(x) = \dfrac{14x - 6x^2}{(1-x)^2(1+3x)}$.

(a) Express $f(x)$ in partial fractions. [4 marks]

Answer .....

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(b) Hence show that, in the expansion of $f(x)$ in ascending powers of x, the coefficient of x^n is $2n + 3 - 3(-3)^n$. [5 marks]

(c) Explain why the expansion cannot be used to find $f\left(\frac{1}{2}\right)$. [3 marks]

Total for X3: 12 marks

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X4 The constants a, b, c and D are such that [10 marks]

$$\frac{2x^3 + ax^2 + bx + c}{(x+1)^2(x-2)} \equiv 2 + \frac{3}{x+1} - \frac{1}{(x+1)^2} + \frac{D}{x-2}.$$

Given also that the curve $y = \dfrac{2x^3 + ax^2 + bx + c}{(x+1)^2(x-2)}$ passes through the origin, find the values of a, b, c and D.

Answer $a = \ldots$, $b = \ldots$, $c = \ldots$, $D = \ldots$

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X5 $g(x) = x^3 - 7x - 6.$

(a) Use the factor theorem to show that $(x + 1)$ is a factor of $g(x)$, and hence write $g(x)$ as a product of three linear factors. [2 marks]

Answer .....

(b) Show that $\dfrac{x^4 - 5x^2 + 6x + 6}{g(x)}$ can be written as $x + \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{x-3}$, and find A, B and C. [6 marks]

Answer $A = \ldots$, $B = \ldots$, $C = \ldots$

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(c) Hence prove that $\dfrac{x^4 - 5x^2 + 6x + 6}{g(x)} > 0$ for all $x > 3$. [3 marks]

Total for X5: 11 marks

X6 In this question a, b and c are real numbers.

(a) Given that a and b are positive, prove by contradiction that $a + b \geq 2\sqrt{ab}$. [4 marks]

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(b) Hence prove that if a, b and c are positive then $(a+b)(b+c)(c+a) \geq 8abc$. [3 marks]

(c) Show, by a counter-example, that the inequality in (b) is not true for all real a, b and c. [3 marks]

Total for X6: 10 marks

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X7 $f(x) = \dfrac{x}{x-2} - \dfrac{8}{x^2-4}$, $x \neq \pm 2$.

(a) Show that $f(x) = \dfrac{x+4}{x+2}$. [3 marks]

(b) Find the two values of k for which the equation $f(x) = k$ has no solution. Give a reason for each. [4 marks]

Answer $k = \ldots$ and $k = \ldots$

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(c) Solve $f(x) = x$. Give your answers in surd form. [3 marks]

Answer .....

Total for X7: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a full proof by contradiction, including parity and remainder arguments.	○	○	○
I can use partial fractions to find a general term of a binomial expansion.	○	○	○
I can find unknown constants in an identity using given conditions.	○	○	○
I can find a counter-example to show a statement is false.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Start with "Assume there is a rational solution $x = \frac{p}{q}$, in its lowest terms".

Hint 2 Multiply through by q^3 to get $p^3 + pq^2 + q^3 = 0$.

Hint 3 p and q cannot both be even. Check the other three cases of odd and even: the left side is always odd.

X2

Hint 1 Every integer is even or odd: write $n = 2k$ or $n = 2k + 1$ and square.

Hint 2 Add two of the possible remainders from (a). Which remainders can $x^2 + y^2$ leave? What does 2027 leave?

Hint 3 (a) and (b) say nothing about numbers with remainder 1. Try small ones: $5, 9, 13, 17, 21$.

X3

Hint 1 $(1 - x)^2$ is a repeated factor, so use $\frac{A}{1 - x} + \frac{B}{(1 - x)^2} + \frac{C}{1 + 3x}$. Put $x = 1$ and $x = -\frac{1}{3}$, then compare the x^2 terms.

Hint 2 Expand each fraction as a binomial series. The coefficient of x^n in $(1 - x)^{-2}$ is $n + 1$.

Hint 3 Each expansion has its own range of x. The whole expansion is valid only where all of them are.

X4

Hint 1 The curve passes through the origin, so the fraction is 0 when $x = 0$. What does that make c?

Hint 2 The right-hand side must also be 0 at $x = 0$. That gives D.

Hint 3 Multiply both sides by $(x + 1)^2(x - 2)$, expand, and compare the x^2 and x coefficients.

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X5

Hint 1 Work out $g(-1)$, then divide $g(x)$ by $(x+1)$ and factorise the quadratic.

Hint 2 Divide $x^4 - 5x^2 + 6x + 6$ by $g(x)$ first. The quotient is x; split the remainder over the three factors.

Hint 3 Only one of the four terms is negative for $x > 3$. Show it is smaller than the term x.

X6

Hint 1 Assume the opposite, $a + b < 2\sqrt{ab}$, and move everything to one side.

Hint 2 Use (a) three times, once for each pair, then multiply the three inequalities.

Hint 3 Multiplying inequalities only works when everything is positive. Try three negative numbers.

X7

Hint 1 Use $x^2 - 4 = (x-2)(x+2)$ as the common denominator, then factorise the top.

Hint 2 Write $\dfrac{x+4}{x+2}$ as $1 + \dfrac{2}{x+2}$. Also remember f is not defined at $x = 2$: what value would f have given there?

Hint 3 Multiply both sides of $\dfrac{x+4}{x+2} = x$ by $x+2$ and use the quadratic formula.

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