A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 10: NUMERICAL METHODS · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-year-2-chapter-10-a-star/

Numerical methods: A* questions

94 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 $\mathrm{f}(x) = x^3 - 3x^2 - 9x + k$, where k is a constant.

(a) Find the set of values of k for which the equation $\mathrm{f}(x) = 0$ has three distinct real roots. [5 marks]

Answer $\cdots < k < \ldots$

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(b) Take $k = 26$. Pam works out $\mathrm{f}(2.5)$ and $\mathrm{f}(3.5)$, finds that both are positive and says: "There is no root of $\mathrm{f}(x) = 0$ between 2.5 and 3.5." By evaluating $\mathrm{f}(3)$, show that Pam is wrong, and state how many roots lie between 2.5 and 3.5. [4 marks]

Answer Number of roots $= \ldots$

(c) Take $k = 27$. Show that $\mathrm{f}(x) = (x - 3)^2(x + 3)$, and explain why the change of sign method cannot be used to locate the root $x = 3$. [4 marks]

Answer Reason: ...

Total for X1: 13 marks

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X2 The equation $2x^3 - 5x - 4 = 0$ has exactly one real root, α. Two rearrangements are suggested: [10 marks]

(I) $x = \dfrac{2x^3 - 4}{5}$ and (II) $x = \sqrt[3]{\dfrac{5x + 4}{2}}$.

By finding the gradient of each rearranged function near α, decide which iteration converges to α. Use it to find α correct to 4 decimal places, justifying the accuracy of your answer.

Answer Converges: ... , $\alpha = \ldots$

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X3 $\mathrm{f}(x) = \dfrac{\ln x}{x} - 0.3$, $x > 0$. The equation $\mathrm{f}(x) = 0$ has exactly two roots, one near 1.6 and one near 6.

(a) The tangent to $y = \mathrm{f}(x)$ at the point where $x = x_n$ meets the x-axis at $x = x_{n+1}$. Show that $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$. [3 marks]

Answer Shown

(b) Show that $\mathrm{f}'(x) = \dfrac{1 - \ln x}{x^2}$. Hence explain why $x_0 = \mathrm{e}$ cannot be used as a starting value. [3 marks]

Answer Reason: ...

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(c) Ana takes $x_0 = 3$, which is nearer the smaller root. Find x_1 and x_2 to 3 decimal places, and explain why the method does not lead to the root nearer x_0. [4 marks]

Answer $x_1 = \ldots$, $x_2 = \ldots$

(d) Find the larger root correct to 3 decimal places, justifying your answer. [4 marks]

Answer Root $= \ldots$

Total for X3: 14 marks

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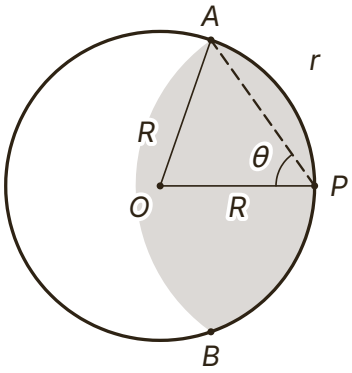
X4 A goat is tied by a rope of length r metres to a post at P on the edge of a circular field with centre O and radius R metres. The rope meets the edge of the field at A and B, and angle $OPA = \theta$ radians. The goat can reach the shaded region. The farmer wants the goat to reach exactly half of the field.

(a) By considering triangle OPA, show that $r = 2R\cos\theta$. [2 marks]



Answer Shown

(b) The area the goat can reach is $r^2\theta + R^2(\pi - 2\theta) - Rr\sin\theta$. Show that θ satisfies $\sin 2\theta - 2\theta\cos 2\theta = \dfrac{\pi}{2}$. [4 marks]

Answer Shown

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(c) Show that the equation in (b) has a root between 0.9 and 1. [2 marks]

Answer Shown

(d) Using the Newton-Raphson method with $\theta_0 = 1$, find θ correct to 4 decimal places. You must justify the accuracy of your answer. [6 marks]

Answer $\theta = \ldots$

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(e) The field has radius 30 m. Find the length of rope needed, to the nearest 0.1 m, and give one reason why the goat might reach more or less than half of the field in practice. [3 marks]

Answer Rope $=$ ... m

Total for X4: 17 marks

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X5 The curves $y = \mathrm{e}^{x/2}$ and $y = 6 - x^2$ meet at two points. Find the x-coordinate of each point, correct to 3 decimal places. You must justify the accuracy of each answer. Solutions relying entirely on calculator technology are not acceptable. [10 marks]

Answer $x = \ldots$ and $x = \ldots$

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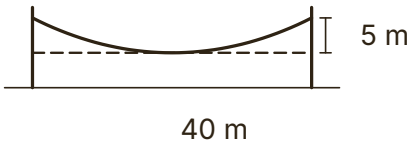
X6 A cable hangs between the tops of two posts 40 m apart. Its lowest point is 5 m below the tops of the posts. A model of its shape leads to the equation $\mathrm{f}(c) = 0$, where

$$\mathrm{f}(c) = c\left(\frac{\mathrm{e}^{20/c} + \mathrm{e}^{-20/c}}{2} - 1\right) - 5, \quad c > 0.$$

(a) Show that $\mathrm{f}(c) = 0$ has a root between 40 and 42. [2 marks]



Answer Shown

(b) Ben rearranges $\mathrm{f}(c) = 0$ to $c = \dfrac{5}{\frac{1}{2}\left(\mathrm{e}^{20/c} + \mathrm{e}^{-20/c}\right) - 1}$ and uses $c_{n+1} = \dfrac{5}{\frac{1}{2}\left(\mathrm{e}^{20/c_n} + \mathrm{e}^{-20/c_n}\right) - 1}$ with $c_0 = 41$. Find c_1 and c_2, and explain why Ben's iteration will not find the root. [4 marks]

Answer $c_1 = \ldots$, $c_2 = \ldots$

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(c) Show that $\mathrm{f}'(c) = \dfrac{\mathrm{e}^{20/c} + \mathrm{e}^{-20/c}}{2} - 1 - \dfrac{10}{c}\left(\mathrm{e}^{20/c} - \mathrm{e}^{-20/c}\right)$. [3 marks]

Answer Shown

(d) Apply the Newton-Raphson method once, with $c_0 = 41$, to find c_1 to 4 decimal places. Show that the root is 40.8 correct to 3 significant figures. [4 marks]

Answer $c_1 = \ldots$

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(e) The model gives the length of the cable as $L = c\left(\mathrm{e}^{20/c} - \mathrm{e}^{-20/c}\right)$ metres. Find L to 3 significant figures, and give one reason why a real cable might not follow the model. [3 marks]

Answer $L =$ ... m

Total for X6: 16 marks

X7 The Newton-Raphson method is used to find $\sqrt{11}$, the positive root of $x^2 - 11 = 0$.

(a) Show that the method gives $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{11}{x_n}\right)$. [2 marks]

Answer Shown

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(b) Show that $x_{n+1} - \sqrt{11} = \dfrac{\left(x_n - \sqrt{11}\right)^2}{2x_n}$. Hence explain why, if $x_0 > 0$, then $x_n \geq \sqrt{11}$ for every $n \geq 1$. [4 marks]

Answer Shown

(c) Starting with $x_0 = 3$, find x_1 and x_2 as fractions. [2 marks]

Answer $x_1 = \ldots$, $x_2 = \ldots$

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(d) Given that $\sqrt{11} > 3.3$, use (b) to show that $0 < x_2 - \sqrt{11} < 0.0002$. [3 marks]

Answer Shown

(e) Writing the method as $x_{n+1} = F(x_n)$, where $F(x) = \frac{1}{2}\left(x + \frac{11}{x}\right)$, find $F'\left(\sqrt{11}\right)$. Explain how this shows why the method converges faster than a typical iteration $x_{n+1} = g(x_n)$ with $0 < |g'(\alpha)| < 1$. [3 marks]

Answer $F'\left(\sqrt{11}\right) = \ldots$

Total for X7: 14 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can say when the change of sign method finds a root and when it misses one.	○	○	○
I can decide whether an iteration converges by finding its gradient at the root.	○	○	○
I can derive and use the Newton-Raphson method and explain when it fails.	○	○	○
I can solve a model numerically, justify the accuracy and criticise the model.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Three distinct roots happen when the maximum point is above the x-axis and the minimum point is below it.

Hint 2 In (b), a curve can go below the axis and come back up between 2.5 and 3.5. Look at the minimum.

Hint 3 In (c), think about the sign of a squared bracket on each side of $x = 3$.

X2

Hint 1 First find two whole numbers that α lies between, so you know roughly where α is.

Hint 2 An iteration $x_{n+1} = g(x_n)$ converges near α when $|g'(\alpha)| < 1$.

For (II), use the fact that at α, $\sqrt[3]{\dfrac{5\alpha + 4}{2}} = \alpha$.

Hint 3 To show the accuracy, evaluate f at the two ends of the interval that rounds to your answer.

X3

Hint 1 In (a), write the equation of the tangent as $y - y_1 = m(x - x_1)$, then put $y = 0$.

Hint 2 In (b), $\ln \mathrm{e} = 1$. What does that make the gradient, and what does a tangent with that gradient look like?

Hint 3 In (c), use your answer to (b): 3 is close to e, so $\mathrm{f}'(3)$ is very small. Dividing by a very small number gives a big step.

X4

Hint 1 In (a), drop a perpendicular from O to PA. It cuts PA in half.

Hint 2 In (b), write everything in terms of R and θ, then use double angle formulae for $\cos^2\theta$ and $\sin\theta\cos\theta$.

Hint 3 In (d), the derivative simplifies: the $2\cos 2\theta$ terms cancel. Keep your calculator in radians.

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X5

Hint 1 Put the two expressions equal and move everything to one side to make one function $\mathrm{f}(x)$.

Hint 2 There are two roots, one positive and one negative. Find whole numbers that each lies between before you start.

Hint 3 For each answer, test f at the two ends of the interval that rounds to it, e.g. 1.8605 and 1.8615.

X6

Hint 1 In (b), the root is between 40 and 41. Are Ben's values getting closer to it?

Hint 2 In (c), $\dfrac{d}{dc}\left(\dfrac{20}{c}\right) = -\dfrac{20}{c^2}$. Use the product rule on the first term.

Hint 3 In (d), to show 40.8 to 3 s.f., test the ends of the interval that rounds to 40.8: 40.75 and 40.85.

X7

Hint 1 In (b), put x_{n+1} over the common denominator $2x_n$ before subtracting $\sqrt{11}$. The top is a perfect square.

Hint 2 In (d), first bound $x_1 - \sqrt{11}$ using $\sqrt{11} > 3.3$, then put that bound into (b).

Hint 3 In (e), compare: for $x = g(x)$, the new error is about $g'(\alpha)$ times the old error. What does (b) say the new error is?

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