A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 11: INTEGRATION · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-year-2-chapter-11-a-star/

Integration: A* questions

83 marks

Date

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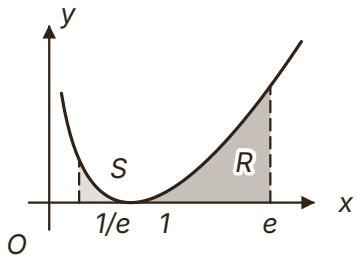
X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The curve C has equation $y = (x - 1)\ln x$, $x > 0$. The region R is bounded by C, the x-axis and the line $x = e$. The region S is bounded by C, the x-axis and the line $x = \dfrac{1}{e}$. Both regions touch the x-axis at $x = 1$.

(a) Show that C never lies below the x-axis. [2 marks]



(b) Show that the area of R is $\dfrac{e^2 - 3}{4}$. [6 marks]

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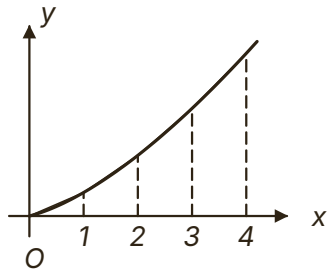
(c) Find the exact area of S, giving your answer in the form $a + \frac{b}{e} + \frac{c}{e^2}$ where a, b and c are rational. [3 marks]

Answer area of $S = \ldots$

Total for X1: 11 marks

X2 The diagram shows the curve $y = x\sqrt{2x+1}$ for $0 \leqslant x \leqslant 4$. The region R is bounded by the curve, the x-axis and the line $x = 4$.

(a) Use the trapezium rule with 4 strips of equal width to estimate the area of R. Give your answer to 2 decimal places. [3 marks]



Answer ...

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(b) Use the substitution $u = 2x + 1$ to find the exact area of R. [5 marks]

Answer area $= \ldots$

(c) Find the percentage error in the estimate in part (a), and use $\dfrac{d^2y}{dx^2}$ to explain why the trapezium rule gives an overestimate here. [4 marks]

Answer $\ldots\%$

Total for X2: 12 marks

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X3 For $x > 0$, y satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{2y}{x(x+2)}$, and $y = 3$ when $x = 1$. Find y in terms of x, and show that $y < 9$ for all $x > 0$. [9 marks]

Answer $y = \ldots$

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X4 The number of fish in a lake, P thousand, t years after a survey is modelled by $\frac{dP}{dt} = \frac{P(10-P)}{20}$. When $t = 0$, $P = 2$.

(a) Show that $P = \frac{10}{1 + 4e^{-\frac{t}{2}}}$. [6 marks]

(b) Find the time it takes for the number of fish to reach 8000. Give your answer as an exact multiple of $\ln 2$ and as a decimal to 3 significant figures. [3 marks]

Answer $t = \ldots$ years

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(c) State the number of fish predicted in the long term, and give one reason why the model may not be suitable. [2 marks]

Answer ...

Total for X4: 11 marks

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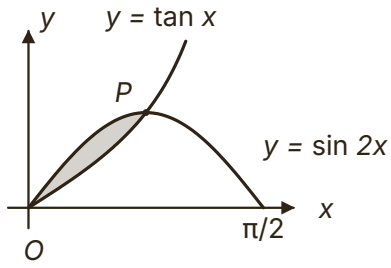
X5 The curves $y = \sin 2x$ and $y = \tan x$, $0 \leqslant x < \dfrac{\pi}{2}$, meet at the origin and at the point P. Find the exact area of the region bounded by the two curves between O and P, giving your answer in the form $a + b \ln 2$. [9 marks]



Answer area $= \ldots$

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X6 $I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\, dx$ and $J = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x}\, dx$.

(a) Use the substitution $u = \dfrac{\pi}{2} - x$ to show that $I = J$. [4 marks]

(b) Find $I + J$, and hence find the exact value of I. [3 marks]

Answer $I = \ldots$

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(c) Hence find the exact value of $\displaystyle\int_0^{\frac{\pi}{2}} \frac{2\sin x + 5\cos x}{\sin x + \cos x}\,dx$. [2 marks]

Answer ...

Total for X6: 9 marks

X7 For $0 \leqslant x \leqslant \dfrac{\pi}{2}$, y satisfies the differential equation $\dfrac{dy}{dx} = xy^2\cos x$, and $y = \dfrac{1}{2}$ when $x = 0$.

(a) Show that $y = \dfrac{1}{3 - x\sin x - \cos x}$. [7 marks]

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(b) Find the exact value of y when $x = \frac{\pi}{2}$. [2 marks]

Answer $y = \ldots$

(c) By considering $g(x) = x \sin x + \cos x$, show that the solution is defined for all x in $0 \leqslant x \leqslant \frac{\pi}{2}$. [3 marks]

Total for X7: 12 marks

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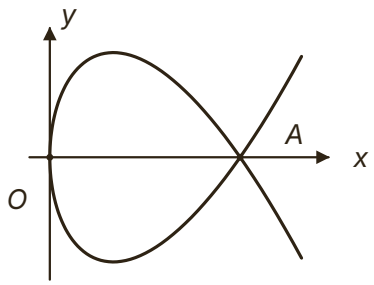
X8 The curve C has parametric equations $x = t^2$, $y = t^3 - 4t$, $t \in \mathbb{R}$. Part of C forms a loop, as shown.



(a) Find the coordinates of the points where C meets the x-axis. [2 marks]

Answer ...

(b) Find the exact area enclosed by the loop. [6 marks]

Answer area $=$ ...

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(c) Rhys works out $\int_{-2}^{2} y\frac{dx}{dt}\,dt$ and gets $-\frac{256}{15}$. He says an area cannot be negative, so he has made a mistake. Explain why his integral is negative, and why its size is still the area of the loop. [2 marks]

Total for X8: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can choose between substitution, parts and partial fractions for an integral.	○	○	○
I can find areas between curves and areas under a curve given in parametric form.	○	○	○
I can use the trapezium rule and explain whether it gives an over- or underestimate.	○	○	○
I can solve a separable differential equation and use the solution in a model.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 A product of two negative numbers is positive. Check the sign of each factor on each side of $x = 1$.

Hint 2 Integrate by parts with $u = \ln x$, because $\ln x$ is easy to differentiate and hard to integrate.

Hint 3 Both regions lie above the axis, so both areas are the antiderivative evaluated in the usual order. Use $\ln \dfrac{1}{e} = -1$.

X2

Hint 1 The strip width is 1. Keep at least 5 significant figures in each y value.

Hint 2 Change everything to u: the x outside the root, the dx and both limits.

Hint 3 Differentiate $\dfrac{3x+1}{(2x+1)^{\frac{1}{2}}}$ with the quotient rule. A convex curve sits below its chords.

X3

Hint 1 Separate the variables: put every y with dy and every x with dx.

Hint 2 Split $\dfrac{2}{x(x+2)}$ into partial fractions before integrating.

Hint 3 Combine the logarithms and remove them. Then divide $9x$ by $x + 2$ to compare y with 9.

X4

Hint 1 Put $\dfrac{20}{P(10-P)}$ into partial fractions. Integrating $\dfrac{1}{10-P}$ gives $-\ln(10-P)$.

Hint 2 Use $t = 0$, $P = 2$ before you remove the logarithms, then make P the subject.

Hint 3 P is in thousands, so 8000 fish is $P = 8$.

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X5

Hint 1 Write $\sin 2x$ as $2\sin x\cos x$ and $\tan x$ as $\dfrac{\sin x}{\cos x}$. Do not divide by $\sin x$: you would lose $x = 0$.

Hint 2 The area is the integral of the top curve minus the bottom curve.

Hint 3 $\tan x = \dfrac{\sin x}{\cos x}$ is $-\dfrac{f'(x)}{f(x)}$ with $f(x) = \cos x$.

X6

Hint 1 Change the limits as well as the integrand. The minus sign from $du = -dx$ swaps the limits back.

Hint 2 I and J have the same denominator, so add the integrands.

Hint 3 Split the fraction into a multiple of I plus a multiple of J.

X7

Hint 1 Divide by y^2 and integrate each side. The right side needs integration by parts.

Hint 2 Use the condition $y = \dfrac{1}{2}$ when $x = 0$ before you make y the subject.

Hint 3 The solution fails only where $3 - g(x) = 0$. Find the largest value of g on the interval.

X8

Hint 1 The curve meets the x-axis where $y = 0$. Solve for t, then find x.

Hint 2 Work out the area of the top half and double it. The top half is where $y \geqslant 0$.

Hint 3 Track which way x moves and which side of the axis the curve is on, for each half of the loop.

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