A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 3: SEQUENCES AND SERIES · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-pure-year-2-chapter-3-a-star/

Sequences and series: A* questions

78 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 An arithmetic series has first term a and common difference d, where $d \neq 0$. The 1st, 4th and 13th terms of the arithmetic series are the first three terms of a geometric series.

(a) Show that $a = \dfrac{3d}{2}$. [3 marks]

(b) Find the common ratio of the geometric series. [2 marks]

Answer .....

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(c) The sum of the first 10 terms of the arithmetic series is 120. Find a and d. [2 marks]

Answer .....

(d) Find the least value of n for which the sum of the first n terms of the geometric series is greater than one million. [3 marks]

Answer .....

Total for X1: 10 marks

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X2 A geometric series has first term a and common ratio r, where $0 < r < 1$. Its sum to infinity is 16. The sum to infinity of its odd-numbered terms, $u_1 + u_3 + u_5 + \ldots$, is $\frac{64}{5}$. Find the sum to infinity of its even-numbered terms, $u_2 + u_4 + u_6 + \ldots$ [9 marks]

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X3 A sequence is defined by $u_1 = 2$ and $u_{n+1} = pu_n + q$, where p and q are constants. $u_2 = 6$, and the sequence converges to 10.

(a) Find p and q. [4 marks]

Answer .....

(b) Show that $u_n = 10 - 8\left(\frac{1}{2}\right)^{n-1}$. [3 marks]

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(c) Hence find the exact value of $\sum_{n=1}^{20} u_n$. [4 marks]

Answer .....

Total for X3: 11 marks

X4 A tree is planted when it is 1.2 m tall. In its first year it grows 0.6 m. In each later year it grows 85% of the amount it grew in the year before.

(a) Find the height of the tree after 5 years, in metres to 3 significant figures. [3 marks]

Answer .....

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(b) Show that the tree never reaches a height of 5.2 m. [2 marks]

(c) Find the year in which the height of the tree first exceeds 5 m. [3 marks]

Answer .....

(d) Give two criticisms of this model. [2 marks]

Total for X4: 10 marks

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X5 [8 marks]

Find the value of $\sum_{r=1}^{99}(-1)^r(3r+1)$.

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X6 The series S is $3 + 6\cos\theta + 12\cos^2\theta + 24\cos^3\theta + \ldots$, where $0 < \theta < \pi$.

(a) Find the range of values of θ for which S has a sum to infinity. [3 marks]

Answer .....

(b) Write down an expression for the sum to infinity in terms of θ. [2 marks]

Answer .....

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(c) Show that, for these values of θ, the sum to infinity is greater than $\frac{3}{2}$. [3 marks]

(d) Given that the sum to infinity is 6, find θ, in radians to 3 decimal places. [3 marks]

Answer .....

Total for X6: 11 marks

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X7 An arithmetic series has first term a and common difference d. The sum of its first 20 terms equals the sum of its 21st to 30th terms inclusive.

(a) Show that $2a = 11d$. [4 marks]

(b) Given also that the 10th term is 29, find a and d. [3 marks]

Answer .....

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(c) Find the greatest value of n for which the sum of the first n terms is less than 2000. [3 marks]

Answer .....

Total for X7: 10 marks

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X8 A geometric sequence has positive terms. Its first term is $u_1 = 6$ and its fourth term is $u_4 = 48$. Find the least value of n for which $u_1 \times u_2 \times \cdots \times u_n > 10^{100}$. [9 marks]

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can link arithmetic and geometric sequences and find unknown terms.	○	○	○
I can find a sum to infinity and the values for which it exists.	○	○	○
I can work with sigma notation, alternating series and recurrence relations.	○	○	○
I can use logarithms to find the least number of terms that meets a condition.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Three numbers p, q, s are consecutive terms of a geometric sequence when $q^2 = ps$.

Hint 2 The 1st, 4th and 13th terms are a, $a + 3d$ and $a + 12d$.

Hint 3 For (d), $S_n = \dfrac{a(r^n - 1)}{r - 1}$ with the geometric first term $a = 3$. Take logarithms of $3^n > \ldots$

X2

Hint 1 The odd-numbered terms $a, ar^2, ar^4, \ldots$ form a geometric series of their own. What is its ratio?

Hint 2 Divide one sum-to-infinity equation by the other, and factorise $1 - r^2$.

Hint 3 The whole series is the odd terms plus the even terms.

X3

Hint 1 At the limit, u_{n+1} and u_n are both 10, so $10 = 10p + q$. Write u_2 in terms of p and q for a second equation.

Hint 2 For (b), look at how far u_n is from 10.

Hint 3 $\sum(10 - 8 \times 0.5^{n-1})$ is 20×10 minus the sum of a geometric series.

X4

Hint 1 The yearly growths $0.6, 0.6 \times 0.85, \ldots$ form a geometric series. The height is 1.2 plus their sum.

Hint 2 The total growth can never be more than the sum to infinity.

Hint 3 Set $1.2 + S_n > 5$, make 0.85^n the subject and take logarithms. Remember the inequality turns round when you divide by $\ln 0.85$.

X5

Hint 1 Write out the first few terms. The signs alternate: $-4 + 7 - 10 + 13 - \ldots$

Hint 2 Group the terms in pairs, starting from the first. What is each pair worth?

Hint 3 There are 99 terms, an odd number, so one term is left over at the end.

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X6

Hint 1 Divide each term by the one before to find the common ratio.

Hint 2 A geometric series converges when $-1 < r < 1$. Solve $-1 < 2\cos\theta < 1$ in the given range.

Hint 3 For (c), find the range of $1 - 2\cos\theta$ first. Its smallest value is above 0.

X7

Hint 1 The sum of the 21st to 30th terms is $S_{30} - S_{20}$.

Hint 2 The 10th term is $a + 9d$. Substitute $a = \dfrac{11d}{2}$.

Hint 3 S_n simplifies to a quadratic in n. Check the whole numbers either side of the root: the inequality is strict.

X8

Hint 1 $u_4 = u_1 r^3$, so find r first.

Hint 2 The log of a product is the sum of the logs, and $\log_2(6 \times 2^{k-1}) = \log_2 6 + (k-1)$ is an arithmetic sequence.

Hint 3 Change 10^{100} to base 2: $\log_2 10^{100} = 100\log_2 10$. Then test whole numbers near the root.

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